Saturday, 02 December 2017
“In the name of Allah, the Most Beneficent, the Most Merciful.”

Narrated Anas (RA): The Prophet(SAW) used to say, “O Allah! Our Lord! Give us in this world that, which is good and in the Hereafter that, which is good and save us from the torment of the Fire” (Al-Bukhari)

Dear Brothers and Sisters!

I kindly request each one of you to remember me, my parents, my children, my family and the entire Muslims of the world in your daily Prayers.

abullahwardak53@gmail.com
LESSON 1

POWER

Example: \(a^n = a \times a \times \ldots \times a\) \((n\ times)\)

Example: \(4^3 = 4 \times 4 \times 4 = 64\)

Example: \(-4^3 = -(4 \times 4 \times 4) = -64\)

Example: \(-4^2 = -(4 \times 4) = -16\)

Example: \(2a^n \times 3a^n = (2 \times 3) \times a^{m+n} = 6a^{m+n}\)

Example: \(2n^4 \times 6n^3 = (2 \times 6) \times n^{4+3} = 12n^7\)

Example: \((3y^2)^3 = 3 \times (y^2)^3 = 27y^6\)

Example: \(4 \times 4 \times 4 = 4^3\) \quad \(2 \times 2 \times 2 \times 2 = 2^5\) \quad \(a \times a \times a = a^3\)

Example: \(3 \times 3 \times 3 = 3^3\) \quad \(5 \times 5 \times 5 \times 5 = 5^3\) \quad \(b \times b \times b = b^3\)

Example: \(-4 \times -4 \times -4 = (-4)^3\) \quad \(-2 \times -2 \times -2 \times -2 = (-2)^4 = 2^4\)

Example: \(-a \times -a \times -a = (-a)^3 = -a^3\) \quad \(-b \times -b = (-b)^2 = b^2\)

Example: \(\left(\frac{a}{b}\right)^5 = \frac{a \times a \times a \times a \times a}{b \times b \times b \times b \times b} = \frac{a^5}{b^5}\)

Example: \(\left(\frac{2}{3}\right)^3 = \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} = \frac{2^3}{3^3} = \frac{8}{27}\)

Example: \(2^2 = 2 \times 2 = 4\) \quad \(3^1 = 3 \times 3 \times 3 = 27\) \quad \(4^3 = 4 \times 4 \times 4 = 64\)

Example: \(a^2 = a \times a\) \quad \(a^3 = a \times a \times a\) \quad \(b^3 = b \times b \times b\)

Example: \(-2^2 = -(2 \times 2) = -4\) \quad \(-3^3 = -(3 \times 3 \times 3) = -27\)

Example: \(-a^2 = -(a \times a)\) \quad \(-a^3 = -(a \times a \times a)\) \quad \(-b^3 = -(b \times b \times b)\)
Example: \((a^n)^m = a^{mn}\)

Example: \((2^3)^2 = 2^2 \times 2^2 = 2 \times 2 \times 2 \times 2 = (2)^4 = 16\)

Example: \((3^3)^2 = 3^2 \times 3^2 = 3 \times 3 \times 3 \times 3 = (3)^4 = 81\)

Example: \((2^3)^2 = 2^3 \times 2^3 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 = (2)^6 = 64\)

Example: \((2^3)^2 = (2)^6 = 2 \times 2 \times 2 \times 2 = 16\)

Example: \((3^3)^2 = (3)^4 = 3 \times 3 \times 3 \times 3 = 81\)

Example: \((2^3)^2 = (2)^6 = 2 \times 2 \times 2 \times 2 \times 2 = 64\)

Example: \(a^{2n} \times a^{3n} = a^{2n+3n} = a^{5n}\)

Example: \(a^n \times a^m = a^{n+m}\)

Example: \((-2)^3 \times (-2)^6 = (-2)^{3+5} = (-2)^8 = 256\)

Example: \(a^{2n+3} \times a^{3n+5} = a^{2n+3+3n+5} = a^{5n+8}\)

Example: \(\frac{2^n}{2^m} = (2)^{n-m}\)

Example: \(\frac{2^3}{2^2} = (2)^{3-2} = 2^1 = 2\) and \(\frac{2^3}{2^2} = \frac{2 \times 2 \times 2}{2 \times 2} = 2\)

Example: \(\frac{2^5}{2^3} = (2)^{5-3} = 2^2 = 2 \times 2 = 4\)

Example: \(\frac{2^5}{2^3} = (2)^{5-(-3)} = 2^{5+3} = 2^8\)

Example: \(\frac{2^{-2}}{1} = \frac{1}{2^2} = \frac{1}{4}\)

Example: \(\frac{2^{-2}}{2^3} = (2)^{-2-3} = 2^{-5} = \frac{1}{2^5} = \frac{1}{32}\)
Example: \[
\frac{2^{-2}}{2^{-3}} = (2)^{-2 - (-3)} = 2^{-2 + 3} = 2^1 = 2
\]

Example: \[
\frac{2^2}{2^1} = (2)^{2 - 1} = 2^1 = 2
\]

Example: \[
\frac{2^2}{2^{-1}} = (2)^{2 + 1} = 2^3
\]

Example: \[
\frac{1}{2^3} = (2)^{-1} = 2^{-3} = 2^4 = 2
\]

Example: \[
\frac{16^4}{16^1} = (16)^{3} = 16^3 = 16^4 = (2 \times 2 \times 2) = (2^4) = 2^4 = 2
\]

Example: \[
\frac{a^3}{a^5} = \frac{(a)^{3-5}}{a} = a^{15 - 12} = a^{20} = a^{20}
\]

Example: \[
\frac{a^3}{a^5} = \frac{(a)^{3-5}}{a} = a^{15 - 12} = a^{20} = a^{20}
\]

Example: \[
\frac{a^{2n}}{a^n} = (a)^{2n-n} = a^n
\]

Example: \[
\frac{a^{2n}}{a^{3m}} = (a)^{2n-3m}
\]

Example: \[
\frac{a^{2x+3}}{a^{x-1}} = (a)^{2x+3-(x-1)} = (a)^{x+4}
\]

Example: \[
\frac{a^{2x}b^{3y}}{a^yb^{2y}} = (a)^{2x-1} \times (b)^{3y-2y} = a^x \times b^y = a^x b^y
\]

Example: \[
(a \times b)^n = a^n \times b^n
\]

Example: \[
(2 \times 3)^2 = 2^2 \times 3^2 = 4 \times 9 = 36
\]
Example: \((2 \times 3)^2 = 6^2 = 6 \times 6 = 36\)

Example: \((-2 \times 3)^2 = (-2)^2 \times 3^2 = 4 \times 9 = 36\)

Example: \((-2 \times 3)^2 = (-6)^2 = (-6) \times (-6) = 36\)

Example: \((2 \times -3)^2 = 2^2 \times (-3)^2 = 4 \times 9 = 36\)

Example: \((-2 \times -3)^2 = (-2)^2 \times (-3)^2 = 4 \times 9 = 36\)

Example: \((-2 \times -3)^2 = 6^2 = 6 \times 6 = 36\)

Example: \(\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n} = \left(\frac{a}{b}\right)^n \quad b \neq 0\)

Example: \(\left(\frac{a}{b}\right)^k = \frac{a^k}{b^k} = \left(\frac{a}{b}\right)^k \quad b \neq 0\)

Example: \(\left(\frac{2}{3}\right)^3 = \frac{2^3}{3^3} = \frac{8}{27}\)

Example: \(\left(\frac{-2}{3}\right)^3 = \frac{(-2)^3}{3^3} = \frac{-8}{27} = \frac{-8}{27}\)

Example: \(\left(\frac{2}{-3}\right)^3 = \frac{2^3}{(-3)^3} = \frac{8}{-27} = \frac{8}{27}\)

Example: \(\left(\frac{-2}{-3}\right)^3 = \frac{(-2)^3}{(-3)^3} = \frac{-8}{-27} = \frac{8}{27}\)

Example: \(\left(\frac{-2}{-3}\right)^3 = \frac{2^3}{3^3} = \frac{2^3}{3^3} = \frac{8}{27}\)

Example: \(a^0 = 1 \quad a \neq 0\)

Example: \(\frac{2^3}{2^3} = \frac{8}{8} = 1\)  \quad \text{Example:} \quad \frac{2^3}{2^3} = 2^{3-3} = 2^0 = 1\)
Example: \( \frac{2^5}{2^5} = \frac{32}{32} = 1 \)  
Example: \( \frac{2^5}{2} = 2^{5-5} = 2^0 = 1 \)

Example: \( \left( \frac{1}{2} \right)^0 = 1 \)  
Example: \( \left( -\frac{1}{2} \right)^0 = 1 \)

Example: \( (-5)^0 = 1 \)  
Example: \( \left( -\frac{1}{2000} \right)^0 = 1 \)

Example: \( (4a)^0 = 1 \quad a \neq 0 \)

Example: \( 4a^0 = 4 \times 1 = 4 \quad a \neq 0 \)

Example: \( -4a^0 = -4 \times 1 = -4 \quad a \neq 0 \)

Example: \( \left(-\frac{x + 7}{2a - 5}\right)^0 = 1 \quad a \neq \frac{5}{2} \)

Example: \( (a^n)^k = a^{kn} = a^{k \times n \times n} \)

Example: \( (a^2)^3 = a^{5 \times 3 \times 2} = a^{30} \)

Example: \( (a^2)^{-3} = a^{5 \times (-3) \times 2} = a^{-30} \)

Example: \( (a^{-2})^3 = a^{-5 \times 3 \times (-2)} = a^{30} \)

Example: \( \left( a^{-\frac{2}{3}} \right)^3 = a^{-5 \times 3 \times \left(-\frac{2}{3}\right)} = a^{30} = a^{24} = a^4 \)

Example: Find the value of \( y \) when \( a = 2 \)

(i) \( y = a^3 - 3a \)  
\text{Ans:} \quad y = 2^3 - 3 \times 2 = 8 - 6 = 2 \)

(ii) \( y = a^{-3} + 5 \)  
\text{Ans:} \quad y = 2^{-3} + 5 = \frac{1}{2^3} + 5 = \frac{1}{8} + 5 = 5 \frac{1}{8} \)
(iii) \( y = \frac{a^2 - 2}{a} \)

Ans: \( y = \frac{a^2 - 2}{a} = \frac{2^2 - 2}{2} = \frac{4 - 2}{2} = \frac{2}{2} = 1 \)

(iv) \( a^3 + 5 = y + 3a \)

Ans: \( y = a^3 - 3a + 5 = 2^3 - 3(2) + 5 = 8 - 6 + 5 = 7 \)

Example Find the value of \( y \) when \( a = -2 \)

(i) \( y = a^3 - 3a \)

Ans: \( y = (-2)^3 - 3 \times (-2) = -8 + 6 = -2 \)

(ii) \( y = a^{-3} + 5 \)

Ans: \( y = (-2)^{-3} + 5 = \frac{1}{(-2)^3} + 5 = \frac{1}{-8} + 5 = 5 - \frac{1}{8} = \frac{40}{8} - \frac{1}{8} = \frac{39}{8} = 4 \frac{7}{8} \)

(iii) \( y = \frac{a^2 - 2}{a} \)

Ans: \( y = \frac{a^2 - 2}{a} = \frac{(-2)^2 - 2}{-2} = \frac{4 - 2}{-2} = -\frac{2}{2} = -1 \)

(iv) \( a^3 + 5 = y + 3a \)

Ans: \( y = a^3 - 3a + 5 = (-2)^3 - 3(-2) + 5 = -8 + 6 + 5 = 3 \)

Example \( \sqrt[n]{a^m} = a^{\frac{m}{n}} \)

Example \( \sqrt{a} = \sqrt[2]{a^{\frac{1}{2}}} = a^{\frac{1}{2}} \)

Example \( \sqrt[3]{a} = \sqrt[3]{a^{\frac{1}{3}}} = a^{\frac{1}{3}} \)

Example \( \sqrt[3]{a^2} = a^{\frac{2}{3}} \)

Example \( \sqrt[5]{a^2} \times \sqrt[3]{a^2} = a^{\frac{2}{5}} \times a^{\frac{2}{3}} = (a)^{\frac{2}{5} + \frac{2}{3}} = a^{\frac{6+10}{15}} = a^{\frac{16}{15}} \)

Example \( \frac{\sqrt[3]{a^2}}{\sqrt[5]{a^2}} = a^{\frac{3}{2}} = (a)^{\frac{3}{2} - \frac{2}{3}} = a^{\frac{6-10}{15}} = a^{-\frac{4}{15}} \)
Example \( a^m = \sqrt[n]{a^m} \)

Example \( a^2 = \frac{1}{2} \sqrt{a} = \sqrt{a} \)

Example \( a^3 = \frac{1}{3} \sqrt{a} = \sqrt[3]{a} \)

Example \( a^2 = \frac{1}{2} \sqrt{a^2} \)

Example \( a^{16} = \frac{16}{16} \sqrt{a^{16}} = \frac{1}{16} \sqrt{a^{16}} \times \frac{1}{16} \sqrt{1} = a^2 \sqrt{a^8} = a \sqrt{a^8} \)

Example \( a^{-\frac{4}{15}} = \frac{1}{a^{15}} = \frac{1}{\sqrt[15]{a^8}} \)

**Example:** Simplify the following expressions where possible.

(a) \( 3a^2 + 5a^2 \)  
(b) \( 3a^3 \times 5a^5 \)

**Ans:**  
(a) \( 3a^2 + 5a^2 = 8a^2 \)  
(b) \( 3a^3 \times 5a^5 = (3 \times 5) \times a^3 \times a^5 = 15a^8 \)
LESSON 2

Significant-Figures (S.F, s.f)

Decimal Point (D.P OR d.p) and Significant Figures (S.F OR s.f)

<table>
<thead>
<tr>
<th>Calculator Answer</th>
<th>5 d.p</th>
<th>4 d.p</th>
<th>3 d.p</th>
<th>2 d.p</th>
<th>1 d.p</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a) 7638.462</td>
<td>7638.462</td>
<td>7638.462</td>
<td>7638.46</td>
<td>7638.5</td>
<td></td>
</tr>
<tr>
<td>(b) 7.93418</td>
<td>7.93418</td>
<td>7.9342</td>
<td>7.93</td>
<td>7.9</td>
<td></td>
</tr>
<tr>
<td>(c) 1062.5649</td>
<td>1062.5649</td>
<td>1062.5649</td>
<td>1062.365</td>
<td>1062.56</td>
<td>1062.6</td>
</tr>
<tr>
<td>(d) 0.0079252</td>
<td>0.00793</td>
<td>0.0079</td>
<td>0.008</td>
<td>0.01</td>
<td>0.0</td>
</tr>
<tr>
<td>(e) 0.0666666</td>
<td>0.06666</td>
<td>0.0667</td>
<td>0.067</td>
<td>0.07</td>
<td>0.1</td>
</tr>
<tr>
<td>(f) 70631.9353</td>
<td>70631.9353</td>
<td>70631.9353</td>
<td>70631.935</td>
<td>70631.94</td>
<td>70631.9</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Calculator Answer</th>
<th>6 S.F</th>
<th>5 S.F</th>
<th>4 S.F</th>
<th>3 S.F</th>
<th>2 S.F</th>
<th>1 S.F</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a) 7638.462</td>
<td>7638.46</td>
<td>7638.5</td>
<td>7638</td>
<td>7640</td>
<td>7600</td>
<td>8000</td>
</tr>
<tr>
<td>(b) 7.93418</td>
<td>7.93418</td>
<td>7.9342</td>
<td>7.93</td>
<td>7.9</td>
<td>8</td>
<td></td>
</tr>
<tr>
<td>(c) 1062.5649</td>
<td>1062.56</td>
<td>1062.6</td>
<td>1063</td>
<td>1060</td>
<td>1100</td>
<td>1000</td>
</tr>
<tr>
<td>(d) 0.0079252</td>
<td>0.0079252</td>
<td>0.0079252</td>
<td>0.007925</td>
<td>0.00793</td>
<td>0.0079</td>
<td>0.008</td>
</tr>
<tr>
<td>(e) 0.0666666</td>
<td>0.0666666</td>
<td>0.066667</td>
<td>0.06667</td>
<td>0.0667</td>
<td>0.067</td>
<td>0.07</td>
</tr>
<tr>
<td>(f) 70631.9353</td>
<td>70631.9</td>
<td>70632</td>
<td>70630</td>
<td>70600</td>
<td>71000</td>
<td>70000</td>
</tr>
<tr>
<td>(g) 0.9999999</td>
<td>1.00000</td>
<td>1.0000</td>
<td>1.000</td>
<td>1.00</td>
<td>1.0</td>
<td>1</td>
</tr>
<tr>
<td>(h) 25.675231</td>
<td>25.6752</td>
<td>25.675</td>
<td>25.68</td>
<td>25.7</td>
<td>26</td>
<td>30</td>
</tr>
<tr>
<td>(i) -0.006735</td>
<td>-0.006735</td>
<td>-0.006735</td>
<td>-0.006735</td>
<td>-0.00674</td>
<td>-0.0067</td>
<td>-0.007</td>
</tr>
</tbody>
</table>

Example: A football match is watched by 56742 people. Write this number correct to the nearest,

(a) 10000   (b) 1000   (c) 10
Ans: (a) 60000 (b) 57000 (c) 56740

Example: Write each of the following numbers correct to 3 significant figures:

(a) 47316   (b) 303971   (c) 20.453   (d) 0.004368
Ans: (a) 47300 (b) 304000 (c) 20.5 (d) 0.00437
Example: State the upper and lower bounds for each of the following quantities and write an inequality for the actual value in each case.

(a) 4 mm to the nearest mm.  
(b) 15 kg to the nearest kg.  
(c) 4.56 m to the nearest cm.

**Ans:** (a) Upper bound = 4.5 mm  
Lower bound = 3.5 mm  
3.5 mm ≤ actual value < 4.5 mm

(b) Upper bound = 15.5 kg  
Lower bound = 14.5 kg  
14.5 kg ≤ actual value < 15.5 kg

(c) Upper bound = 4.565 m  
Lower bound = 4.555 m  
4.555 m ≤ actual value < 4.565 m

Example: Round each of the following numbers to the nearest 100:

(a) 108  (b) 199  (c) 3471  (d) 59  (e) 33  (f) 451

**Ans:** (a) 100  (b) 200  (c) 3500  (d) 100  (e) 0  (f) 500

Example: Round the number 4765173 to the nearest:

(a) million,  (b) 10,  (c) 1000,  (d) 100.

**Ans:** (a) 5000000  (b) 4765170  (c) 4765000  (d) 4765200

Example: By rounding all numbers to 1 s.f, calculate the estimated value of the following numbers.

(a) \[
\frac{394 \times 215}{4172}
\]  
(b) \[50.5 \times 69.8\]  
(c) \[85.4 \div 25.8\]

(d) \[
\frac{34.5 \times 24.8}{15.0 \times 2.8}
\]  
(e) \[
\frac{44.5 + 23.8}{6.5 - 2.4}
\]  
(f) \[
\frac{520.4 \times 8.065}{99.53}
\]

**Ans:** (a) 394 \approx 400 \ (1 \ s.f)  
(b) 50.5 \approx 50 \ (1 \ s.f)  
(c) 85.4 \approx 90 \ (1 \ s.f)

\[
\frac{394 \times 215}{4172} \approx \frac{400 \times 200}{4000} = 20
\]

(b) 50.5 \approx 50 \ (1 \ s.f)  
69.8 \approx 70 \ (1 \ s.f)

\[
50.5 \times 69.8 \approx 50 \times 70 = 3500
\]

(c) 85.4 \approx 90 \ (1 \ s.f)  
25.8 \approx 30 \ (1 \ s.f)

\[
85.4 \div 25.8 = \frac{85.4}{25.8} \approx \frac{90}{30} = 3
\]
Example: By rounding all numbers to 2 s.f, calculate the estimated value of the following numbers.

(a) \(\frac{384 \times 215}{4372}\) (b) \(50.5 \times 69.8\) (c) \(85.4 \div 25.8\)

(d) \(\frac{34.5 \times 24.8}{15.0 \times 2.8}\) (e) \(\frac{44.5 + 23.8}{6.5 - 2.4}\) (f) \(\frac{520.4 \times 8.065}{99.53}\)

Example: By rounding all numbers to 2 s.f, calculate the estimated value of the following numbers.

(a) \(\frac{384 \times 215}{4372}\) (b) \(50.5 \times 69.8\) (c) \(85.4 \div 25.8\)

(d) \(\frac{34.5 \times 24.8}{15.0 \times 2.8}\) (e) \(\frac{44.5 + 23.8}{6.5 - 2.4}\) (f) \(\frac{520.4 \times 8.065}{99.53}\)

Ans: (a) \(\frac{394 \times 215}{4172}\) \(394 \approx 390\) (2 s.f) \(215 \approx 220\) (2 s.f)

\(4172 \approx 4200\) (2 s.f) \(\Rightarrow \frac{384 \times 215}{4372} \approx \frac{380 \times 220}{4400} = 19\)

Ans: (b) \(50.5 \times 69.8\) \(50.5 \approx 51\) (2 s.f) \(69.8 \approx 70\) (2 s.f)

\(\Rightarrow 50.5 \times 69.8 \approx 51 \times 70 = 3570\)

Ans: (c) \(89.5 \div 29.8\) \(89.5 \approx 90\) (2 s.f) \(29.8 \approx 30\) (2 s.f)

\(\Rightarrow 89.5 \div 29.8 = \frac{89.5}{29.8} \approx \frac{90}{30} = 3\)

Ans: (d) \(\frac{34.5 \times 24.8}{4.98 \times 6.95}\) \(34.5 \approx 35\) (2 s.f) \(24.8 \approx 25\) (2 s.f)
Example: By rounding all numbers to 3 s.f, calculate the estimated value of the following numbers.

(a) \[0.1245 \times 99.95\]  
(b) \[804.5 \div 99.99\]

(c) \[\frac{120.4 \times 49.95}{4.995 \times 9.996}\]  
(d) \[\frac{550.49 + 49.95}{210.58 - 10.955}\]

**Ans:**  
(a) \[0.1245 \times 99.95\] \[0.1245 \approx 0.125 \text{ (3 s.f)}\] \[99.95 \approx 100 \text{ (3 s.f)}\]  
\[\Rightarrow 0.1245 \times 99.95 \approx 0.125 \times 100 = 12.5\]

(b) \[804.5 \div 99.99\] \[804.5 \approx 805 \text{ (3 s.f)}\] \[99.99 \approx 100 \text{ (3 s.f)}\]  
\[\Rightarrow 804.5 \div 99.99 = \frac{804.5}{99.99} \approx \frac{805}{100} = 8.05\]

(c) \[\frac{120.4 \times 49.95}{4.995 \times 9.996}\] \[120.4 \approx 120 \text{ (3 s.f)}\] \[49.95 \approx 50.0 \text{ (3 s.f)}\]  
\[4.995 \approx 5.00 \text{ (3 s.f)}\] \[9.996 \approx 10.0 \text{ (3 s.f)}\]  
\[\Rightarrow \frac{120.4 \times 49.95}{4.995 \times 9.996} = \frac{120 \times 50.0}{5 \times 10} = 120\]

(d) \[\frac{550.49 + 49.95}{210.58 - 10.955}\] \[550.49 \approx 550 \text{ (3 s.f)}\] \[49.95 \approx 50.0 \text{ (3 s.f)}\]
Example: By rounding all numbers to 1 s.f, calculate the estimated value of the following numbers.

(a) \( \frac{40.68 + 61.2}{9.96 \times 5.13} \)  
(b) \( \frac{3.87 \times 5.07^3}{5.16 \times 19.87} \)

Ans: (a) \( \frac{40.68 + 61.2}{9.96 \times 5.13} \approx 40 \) (1 s.f)  
Ans: (b) \( \frac{3.87 \times 5.07^3}{5.16 \times 19.87} \approx 5 \) (1 s.f)

Note: Using the calculator, the actual answer is 1.9939407... so the answer is a good approximation.

Ans: (b) \( \frac{3.87 \times 5.07^3}{5.16 \times 19.87} \approx 5 \) (1 s.f)  
\( 5.16 \approx 5 \) (1 s.f)  
\( 19.87 \approx 20 \) (1 s.f)  
\( \Rightarrow \frac{3.87 \times 5.07^3}{5.16 \times 19.87} \approx \frac{4 \times 125}{5 \times 20} = \frac{500}{100} = 5 \) (This is correct)

Note: \( 5.07^3 = (5.07)^3 = 130.32384 \approx 100 \) (Do not do it like this!)

Ans: (c) \( \frac{2.78 + \pi}{\sqrt{5.95 \times 6.32}} \)  
\( 2.78 \approx 3 \) (1 s.f)  
\( \pi \approx 3.14159 \approx 3 \) (1 s.f)  
\( \sqrt{5.95} \approx \sqrt{6} \) (1 s.f)  
\( 6.32 \approx 6 \) (1 s.f)  
\( \Rightarrow \frac{2.78 + \pi}{\sqrt{5.95 \times 6.32}} \approx \frac{3 + 3}{\sqrt{6} \times 6} = \frac{6}{6} = 1 \)

Ans: (d) \( \frac{59.96}{40.21 + 19.86} + \sqrt{8.652} \)  
\( 59.96 \approx 60 \) (1 s.f)  
\( 40.21 \approx 40 \) (1 s.f)  
\( 19.86 \approx 20 \) (1 s.f)  
\( \sqrt{8.652} \approx 3 \) (1 s.f)  
\( \Rightarrow \frac{59.96}{40.21 + 19.86} + \sqrt{8.652} \approx \frac{60}{40 + 20} + 3 = 4 \)

Example: The length of an envelope is 21 cm to the nearest cm. What is the smallest possible real length of the envelope?

Ans: 20.5 cm
LESSON 3

RATIO

Example: In a box there are 8 apples and 12 oranges. Write this as a ratio comparing the number of apples to the number of oranges.

Ans: The ratio of the number of apples to the number of oranges is 8 to 12. You write this as 8:12.

OR The ratio of the number of oranges to the number of apples is 12 to 8. You write this as 12:8.

Note: The order is important in ratios.

Equivalent Ratios

Equivalent ratios are ratios that are equal to each other.

Example: The following ratios are all equivalent to 2 : 5. 
2 : 5 = 6 : 15 = 8 : 20 = 10 : 25 = 20 : 50 = ...

Example: 1 : 2 = 2 : 4
Example: 4 : 7 = 12 : 21
Example: 16 : 24 = 2 : 3
Examples: 0.5 : 5 = 1 : 10
Examples: 0.2 : 3 = 2 : 30.
Examples: 12.5 : 15 = 125 : 150 = 5 : 6

Examples: \( \frac{3}{11} : \frac{4}{11} = 3 : 4 \) i.e. multiply each side by 11 to get 3 : 4

Examples: \( \frac{3}{4} : \frac{4}{5} = 15 : 16 \) i.e. multiply each side by 20 to get 15 : 16

Examples: \( \frac{2}{3} : \frac{2}{7} = 14 : 6 = 7 : 3 \) i.e. multiply each side by 21 to get 7 : 3
Example: Write the following ratios in their simplest form.
(a ) $10 : 15$  (b ) $121 : 44$  (c ) $\frac{4}{3} : \frac{1}{4}$  (d ) $2.5 : 0.5$
Ans: (a ) $2 : 3$  (b ) $11 : 4$  (c ) $16 : 1$  (d ) $5 : 1$

Example: Express the ratio of 40p to £2.
Ans: $40 : 200 = 1 : 5$

Example: Express the ratio of 5 km to 500 m in its simplest form.
Ans: $5000 : 500 = 10 : 1$

Example: Express the ratio $\frac{1}{3} : \frac{1}{4}$ in its simplest form.
Ans: $4 : 3$ by multiplying both sides by 12

Example: Express the ratio $\frac{3}{5} : \frac{5}{7}$ in its simplest form.
Ans: $21 : 25$ by multiplying both sides by 35

Example: Divide £60 between two brothers in the ratio 3 : 2. How much does each get?
Ans: Number of parts = $3 + 2 = 5$
Value of each part = $\frac{\£60}{5} = \£12$
The first brother receives = $3 \times \£12 = \£36$
The second brother receives = $2 \times \£12 = \£24$

Example: Two salaries are in the ratio 3.5 : 4. If the first salary is £3500, what is the second salary?
Ans: Amount of one portion of salary = $\frac{\£3500}{3.5} = \£1000$
The 2$^{nd}$ salary is $4 \times \£1000 = \£4000$

Example: Two cities with populations of 45000 and 25000 receive a grant for £28000. The government decided to share the money in proportion to the population. How much does each city get?
Ans: ratio = $45000 : 25000 = 45 : 25 = 9 : 5$
Total number of shares = $9 + 5 = 14$
Amount of one share = $\frac{\£28000}{14} = \£2000$
The 1$^{st}$ city receives $9 \times \£2000 = \£18000$
The 2$^{nd}$ city receives $5 \times \£2000 = \£10000$
Example: Divide £1344 between Ahmad, Omar and Ismail in the ratio 7 : 5 : 9 respectively. How much does each person receive?

Method

1. Find the total number of shares.
2. Find the amount of one share
3. Find the amount each receives.

Calculation

1. Total number of shares = 7 + 5 + 9 = 21
2. Amount of one share = \( \frac{£1344}{21} = £64 \)
3. Ahmad receives 7 \( \times \) 64 = £448
4. Omar receives 5 \( \times \) 64 = £320
5. Ismail receives 9 \( \times \) 64 = £576
Check: £448 + £320 + £576 = £1344

Note: If we want to get 7:5:9

Example: In a gathering of Afghan tribal elders there was a mix of educational background. The doctors, the engineers and the lawyers were in the ratio of 1.5:2.5:4.5
If there were 300 doctors in the gathering.
How many lawyers were there in the gathering?
How many Engineers were there in the gathering?

Ans: Number in one portion = \( \frac{300}{1.5} = 200 \)

Lawyers = 4.5 \times 200 = 900

Engineers = 2.5 \times 200 = 500

Check: Total = 900 + 500 + 300 = 1700

Note: If we want to get 1.5:4.5:2.5
300:900:500 = 3:9:5 = 1.5:4.5:2.5

Example: Complete the following ratios:

(a) \( 3 : 4 = 6 : ? \)
(b) \( 240 : 400 = ? : 1 \)
(c) \( 18 : 9 = ? : 1 \)
(d) \( 20 : 1 = 64 : ? \)
(e) \( ? : 1 = 12 : 10 \)
(f) \( 1 : ? = 5 : 3 \)

Ans:

\[
\begin{align*}
(a) & \quad 3 : 4 = 6 : ? \\
& \quad \times 2 \\
& \quad 3 : 4 = 6 : 8 \\
& \quad \times 2 \\
(b) & \quad 240 : 400 = ? : 1 \\
& \quad \div 400 \\
& \quad 240 : 400 = 0.6 : 1 \\
& \quad \div 400 \\
(c) & \quad 18 : 9 = ? : 1 \\
& \quad \div 9 \\
& \quad 18 : 9 = 2 : 1 \\
& \quad \div 9 \\
(d) & \quad 20 : 1 = 64 : ? \\
& \quad \times \frac{64}{20} \\
& \quad 20 : 1 = 64 : 3.2 \\
& \quad \times \frac{64}{20}
\end{align*}
\]
Example: For Eid, Abdullah and Abdul Rahman are given money in the ratio of their ages. When Abdullah is 30 years old, Abdul Rahman is 20.

(a) Express their ages as a ratio.

(b) If Abdullah is given £18, how much does Abdul Rahman receive?

(c) Five years later, Abdul Rahman will be given £15. How much will Abdullah receive?

(d) Five years ago, Abdullah was given £25. How much Abdul Rahman would have received?

Ans:

(a) Abdullah : Abdul Rahman = 30 : 20 = 3 : 2

OR Abdul Rahman : Abdullah = 20 : 30 = 2 : 3

(b) Abdul Rahman receives \( \frac{18}{3} \times 2 = £12 \)

(c) Abdullah : Abdul Rahman = (30 + 5) : (20 + 5) = 35 : 25 = 7 : 5

Abdullah receives \( \frac{15}{5} \times 7 = £21 \)

(d) Abdullah : Abdul Rahman = (30 - 5) : (20 - 5) = 25 : 15 = 5 : 3

Abdul Rahman would have received \( \frac{25}{5} \times 3 = £15 \)

Example: In a gathering of Afghan tribal elders there was a mix of educational background. The doctors, the engineers and the lawyers were in the ratio of 2: 3: 5

It was estimated that there were 300 doctors in the gathering. How many lawyers were there in the gathering?
Ans:

Number of ONE portion = \( \frac{300}{2} = 150 \)

No of Lawyers = \( 150 \times 5 = 750 \)
No of Engineers = \( 150 \times 3 = 450 \)
No of Doctors = \( 150 \times 2 = 300 \)
Total = \( 750 + 450 + 300 = 1500 \)

Example: In a gathering of Afghan tribal elders there was a mix of educational background. The doctors, the engineers and the lawyers were in the ratio of 1.5: 2.5: 4.5

It was estimated that there were 300 doctors in the gathering. How many lawyers were there in the gathering?

Ans:

Number of ONE portion = \( \frac{300}{1.5} = 200 \)

No of Lawyers = \( 200 \times 4.5 = 900 \)
No of Engineers = \( 200 \times 2.5 = 500 \)
No of Doctors = \( 200 \times 1.5 = 300 \)
Total = \( 900 + 500 + 300 = 1700 \)

Example: In a gathering of Afghan tribal elders there are doctors, engineers and lawyers. The doctors made up \( \frac{1}{5} \) of the gathering, the engineers made up \( \frac{2}{3} \) of the gathering. It was estimated that there were 300 lawyers in the gathering.

How many doctors and engineers were there in the gathering?

Ans:

Lawyers made up \( 1 - \left( \frac{1}{5} + \frac{2}{3} \right) = 1 - \frac{3 + 10}{15} = 1 - \frac{13}{15} = \frac{15 - 13}{15} = \frac{2}{15} \)

Number of ONE portion = Total number in the gathering
\( = 300 \div \frac{2}{15} = 300 \times \frac{15}{2} = 2250 \)

Engineers = \( 2250 \times \frac{2}{3} = 1500 \)
Doctors = \( 2250 \times \frac{1}{5} = 450 \)

Total = \( 300 + 1500 + 450 = 2250 \)

**OR**

Ratio of doctors, engineers, lawyers = \( \frac{1}{5} : \frac{2}{3} : \frac{2}{15} \)

Multiply each ratio by 15

Ratio of doctors, engineers, lawyers = \( \frac{15}{5} : \frac{30}{3} : \frac{30}{15} \)

Ratio of doctors, engineers, lawyers = \( 3 : 10 : 2 \)

Number of ONE portion = \( \frac{300}{2} = 150 \)

Engineers = \( 150 \times 10 = 1500 \)

Doctors = \( 150 \times 3 = 450 \)

Total = \( 300 + 1500 + 450 = 2250 \)

**Example:**

5 schools sent some students to a conference.

One of the schools sent both boys and girls.

This school sent 16 boys.

The ratio of the number of boys it sent to the number of girls it sent was 1 : 2

The other 4 schools sent only girls.

Each of the 5 schools sent the same number of students.

Work out the total number of students sent to the conference by these 5 schools.

**Ans:** 240
LESSON 4

**Percentage %**

<table>
<thead>
<tr>
<th>Loss</th>
<th>Cost</th>
<th>Profit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Less than 100%</td>
<td>100%</td>
<td>More than 100%</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Selling-Price (S.P)</th>
<th>Buying-Price (B.P)</th>
<th>Selling-Price (S.P)</th>
</tr>
</thead>
<tbody>
<tr>
<td>88% Price</td>
<td>12% Loss</td>
<td>100% Price</td>
</tr>
<tr>
<td></td>
<td>12% Profit</td>
<td>112% Price</td>
</tr>
</tbody>
</table>

**Example No.1:** Someone bought an item for £12 and he made a profit of 13%. What would be the selling price?

**Ans:** Price of 1% = 12/100, then price of 112% = (12/100) x 113 = £13.56

**Example No.2:** Someone bought an item for £12 and he made a loss of 15%. What would be the selling price?

**Ans:** Price of 1% = 12/100, then price of 85% = (12/100) x 85 = £10.2

**Example No.3:** Someone sold an item for £12 and he made a loss of 15%. What would be the buying price?

**Ans:** Price of 1% = 12/85, then price of 100% = (12/85) x 100 = £14.12 (2.d.p)

**Example No.4:** Someone sold an item for £12 and he made a profit of 17%. What would be the buying price?

**Ans:** Price of 1% = 12/117, then price of 100% = (12/117) x 100 = £10.26 (2.d.p)

**Example:** A person bought an item for £5 and sold it with a profit of 30%. How much did he sell the item?

**Ans:** Price of 1% = \( \frac{5}{100} = £0.05 \)

**Price of 130% = Price he sold the item** = £0.05 x 130 = £6.5
Example: A person bought an item for £5 and sold it with a loss of 40%. How much did he sell the item?

Ans: 

\[
\text{Price of } 1\% = \frac{5}{100} = £0.05 \\
\text{Price of 60\% = Price he sold the item} = £0.05 \times 60 = £3
\]

Example: A person sold an item for £220 and made a profit of 10%. How much did he buy the item?

Ans: 

\[
\text{Price of } 1\% = \frac{220}{100 + 10} = \frac{220}{110} = £2 \\
\text{Price of 100\% = Price he bought the item} = 2 \times 100 = £200
\]

Example: A person sold an item for £27 and made a loss of 25%. How much did he buy the item?

Ans: 

\[
\text{Price of } 1\% = \frac{27}{100 - 25} = \frac{27}{75} = £0.36 \\
\text{Price of 100\% = Price he bought the item} = £0.36 \times 100 = £36
\]

Example: Find the percentage profit if a person bought an item for £10 and then sold it for £12.

Ans: 

\[
\text{Profit} = 12 - 10 = £2 \\
\% \text{ Profit} = \frac{2}{10} \times 100 = 20\%
\]

Example: Find the percentage loss if a person bought an item for £12 and then sold it for £10.

Ans: 

\[
\text{Loss} = 12 - 10 = £2 \\
\% \text{ Loss} = \frac{2}{12} \times 100 = 16.667\%
\]

Example: This year a local school has 840 students. Next year this number is expected to increase by 15%. How many students do they expect to have next year? How many more classes will this give in the school if each class has 21 students?

Ans: 

\[
\text{Expected Number of } 1\% \text{ students} = \frac{840}{100} \\
\text{Expected Number of 115\% students} = \frac{840}{100} \times 115 = 966 \\
\text{Expected number of classes} = \frac{966}{21} = 46
\]

Example: A second hand car dealer buys a car for £4225 and sells it at a profit of 20%. Find the selling price?

Ans: 

\[
\text{Price of } 1\% = \frac{4225}{100} = £42.25 \\
\text{Price of 120\% = Price he sold the item} = £42.25 \times 120 = £5070
\]
Example: Ahmad buys a motorcycle for £1250 and sells it for £825. Find his percent loss or %loss?

Ans: Loss = £1250 - £825 = £425, \[ \%\text{Loss} = \frac{425}{1250} \times 100 = 34\% \]

Example: Omar sees a suit he likes in a shop window marked £56. In a sale all prices are reduced by 15%. How much would he saved if he waited for the sale?

Ans: Amount saved = \( \frac{15}{100} \times 56 = £8.4 \), so Omar would have paid 56 - 8.4 = £47.6

OR \[ \text{Price of } 1\% = \frac{56}{100} = £0.56, \text{ Price of } 85\% = £0.56 \times 85 = £47.6 \]

Example: Zaynab buys a stamp for 64p and is able to sell it 6 years later for 80p. Find her percentage profit?

Ans: Profit = 80 - 64 = 16p \[ \%\text{Profit} = \frac{16}{64} \times 100 = 25\% \]

Example: Fatima buys a stamp for 80p and she sold it 6 years later for 64p. Find her percentage loss?

Ans: Loss = 80 - 64 = 16p \[ \%\text{Loss} = \frac{16}{80} \times 100 = 20\% \]

Example: Zaynab buys a stamp for 64p and is able to sell it 6 years later for 128p. Find her percentage profit?

Ans: Profit = 128 - 64 = 64p \[ \%\text{Profit} = \frac{64}{64} \times 100 = 100\% \]
LESSON 5

Scientific Notation (also called Standard Index Form in Britain)

Example: Express 79800 in standard form.
Ans: \(79800 = 7.98 \times 10^4\)

Example: Express 79800.054 in standard form.
Ans: \(79800.054 = 7.9800054 \times 10^4\)

Example: Express \(3.51 \times 10^{-3}\) as an ordinary number.
Ans: \(3.51 \times 10^{-3} = 0.00351\)

Example: Express \(3.5187 \times 10^{-4}\) as an ordinary number.
Ans: \(3.5187 \times 10^{-4} = 0.00035187\)

Example: Here are some more examples

<table>
<thead>
<tr>
<th>Conventional Number</th>
<th>Number in scientific notation or standard index form</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00162</td>
<td>(1.62 \times 10^{-3})</td>
</tr>
<tr>
<td>3742000000000000</td>
<td>(3.742 \times 10^{15})</td>
</tr>
<tr>
<td>816000</td>
<td>(8.16 \times 10^5)</td>
</tr>
<tr>
<td>0.000034</td>
<td>(3.4 \times 10^{-5})</td>
</tr>
<tr>
<td>-231</td>
<td>(-2.31 \times 10^2)</td>
</tr>
<tr>
<td>10</td>
<td>(1 \times 10^1)</td>
</tr>
<tr>
<td>1</td>
<td>(1 \times 10^0)</td>
</tr>
<tr>
<td>0</td>
<td>(0 \times 10^0)</td>
</tr>
</tbody>
</table>

Example: Multiply \((2.7 \times 10^4)\) by \((5 \times 10^{-2})\)
Ans: \((2.7 \times 10^4) \times (5 \times 10^{-2}) = 13.5 \times 10^{(4-2)} = 13.5 \times 10^2 = 1.35 \times 10^3\)

Example: Multiply \((3 \times 10^2)\) by \((2 \times 10^2)\)
Ans: \((3 \times 10^2) \times (2 \times 10^2) = 6 \times 10^{(2+2)} = 6 \times 10^4\)
Example: Evaluate the following, give your answer in standard index form:
\((13.2 \times 10^{-2}) \times (20 \times 10^{-4})\)

Ans: \((13.2 \times 10^{-2}) \times (20 \times 10^{-4}) = (132 \times 10^{-3}) \times (2 \times 10^{-3}) = (264 \times 10^{-6}) = 2.64 \times 10^{-4}\)

Example: Evaluate the following, give your answer in standard index form:
\((1.32 \times 10^{-1}) + (2.2 \times 10^{-3})\)

Ans: 
\[
\frac{1.32 \times 10^{-1}}{2.2 \times 10^{-3}} = \frac{132 \times 10^{-3}}{22 \times 10^{-4}} = 6 \times 10^{(-3+4)} = 6 \times 10^1
\]

Example: Evaluate the following, give your answer in standard index form:
\((13.2 \times 10^{-2}) \div (22 \times 10^{-4})\)

Ans: 
\[
\frac{13.2 \times 10^{-2}}{22 \times 10^{-4}} = \frac{132 \times 10^{-3}}{22 \times 10^{-4}} = 6 \times 10^{(-3+4)} = 6 \times 10^1
\]

Example: Evaluate the following, give your answer in standard index form:
\((5.3 \times 10^3) + (8.2 \times 10^3)\)

Ans: \((5.3 \times 10^3) + (8.2 \times 10^3) = (5.3 + 8.2) \times 10^3 = 13.5 \times 10^3 = 1.35 \times 10^4\)

Example: Evaluate the following, give your answer in standard index form:
\((53 \times 10^2) + (820000 \times 10^{-2})\)

Ans: 
\[
(53 \times 10^2) + (820000 \times 10^{-2}) = (53 + 820000) \times 10^{-2} = 820053 \times 10^{-2} = 8.20053 \times 10^3
\]

Example: Evaluate the following, give your answer in standard index form:
\((5.3) + (8.2 \times 10^3)\)

Ans: \((5.3) + (8.2 \times 10^3) = (0.0053 + 8.2) \times 10^3 = 8.2053 \times 10^3 = 8.2053 \times 10^3\)

Example: Evaluate the following, give your answer in standard index form:
\((5.3 \times 10^3) - (8.2 \times 10^3)\)

Ans: 
\[
(5.3 \times 10^3) - (8.2 \times 10^3) = (5.3 - 8.2) \times 10^3 = -2.9 \times 10^3 = -2.9 \times 10^3
\]

Example: Evaluate the following, give your answer in standard index form:
\((3.3 \times 10^4) - (4.6 \times 10^3)\)

Ans: 
\[
(3.3 \times 10^4) - (4.6 \times 10^3) = (33 \times 10^3) - (4.6 \times 10^3) = 28.4 \times 10^3 = 2.84 \times 10^4
\]

Example: Evaluate the following, give your answer in standard index form:
\((5.3) + (8.2 \times 10^{-2})\)

Ans: 
\[
(530 \times 10^{-2}) + (8.2 \times 10^{-2}) = (530 + 8.2) \times 10^{-2} = 538.2 \times 10^{-2} = 5.382 \times 10^0
\]

Example: Evaluate the following, give your answer in standard index form:
\((5.3 \times 10^4) - (8.2 \times 10^3)\)

Ans: 
\[
(53 \times 10^4) - (8.2 \times 10^3) = (53 - 8.2) \times 10^3 = 44.8 \times 10^3 = 4.48 \times 10^3
\]
Example: Evaluate the following, give your answer in standard index form:

\[(3.3 \times 10^2) - (8.2 \times 10^3)\]

Ans: \((3.3 \times 10^2) - (8.2 \times 10^3) = (0.33 \times 10^3) - (8.2 \times 10^3) = (0.33 - 8.2) \times 10^3 = -7.67 \times 10^3\)

Example: If \( p = 3.5 \times 10^3 \) and \( q = 2 \times 10^2 \). Work out the value of

(a) \( p + q \)  
(b) \( p - q \)  
(c) \( p \times q \)  
(d) \( p \div q \)

Expressing the answer in standard form.

Ans:  
(a) \(3.5 \times 10^3 + 2 \times 10^2 = 35 \times 10^2 + 2 \times 10^2 = (35 + 2) \times 10^2 = 37 \times 10^2 = 3.7 \times 10^3\)

(b) \(3.5 \times 10^3 - 2 \times 10^2 = 35 \times 10^2 - 2 \times 10^2 = (35 - 2) \times 10^2 = 33 \times 10^2 = 3.3 \times 10^3\)

(c) \( (3.5 \times 10^3) \times (2 \times 10^2) = 2 \times 3.5 \times 10^5 = 7 \times 10^5\)

(d) \( (3.5 \times 10^3) \div (2 \times 10^2) = \frac{3.5 \times 10^3}{2 \times 10^2} = 1.75 \times 10^1\)

Example: If \( p = 3.5 \times 10^a \) and \( q = 0.02 \times 10^{a+2} \). Work out the value of

(a) \( p + q \)  
(b) \( p - q \)  
(c) \( p \times q \)  
(d) \( p \div q \)

Expressing the answer in standard form.

Ans:  
(a) \(3.5 \times 10^a + 2 \times 10^2 \times 10^a = 3.5 \times 10^a + 200 \times 10^a = (3.5 + 200) \times 10^a = 203.5 \times 10^a = 2.035 \times 10^{a+2}\)

(b) \(3.5 \times 10^a - 2 \times 10^2 \times 10^a = 3.5 \times 10^a - 200 \times 10^a = (3.5 - 200) \times 10^a = -196.5 \times 10^a = -1.965 \times 10^{a+2}\)

(c) \( (3.5 \times 10^a) \times (200 \times 10^{a+2}) = 700 \times 10^{2a+2} = 700 \times 10^2 \times 10^{2a} = 70000 \times 10^{2a} = 7.0 \times 10^{2a+4}\)

(d) \( (3.5 \times 10^a) \div (2 \times 10^{a+2}) = \frac{3.5 \times 10^a}{2 \times 10^{a+2}} = 1.75 \times 10^{-2}\)
LESSON 6

Multiplying out Brackets

Example: Simplify \(2(4x + 6)\).

Ans: Multiply THE TERMS.

\[
2(4x + 6) = 8x + 12
\]

Example: Simplify \(3q(5r - 4q)\).

Ans: Multiply THE TERMS.

\[
3q(5r - 4q) = 15qr - 12q^2
\]

Example: Simplify \(-5(2a - 3b)\).

Ans: Multiply THE TERMS.

\[
-5(2a - 3b) = -10a + 15b
\]
Example: Simplify \((2a - 5)(3a + 1)\).

**Ans:** Multiply THE TERMS

\[
(2a - 5)(3a + 1) = 6a^2 + 2a - 15a - 5 = 6a^2 - 13a - 5
\]

Example: Simplify \((5t + 3)(5t + 3)\).

**Ans:** Multiply THE TERMS

\[
(5t + 3)(5t + 3) = 25t^2 + 15t + 15t + 9 = 25t^2 + 30t + 9
\]

Example: Factorise \(6x^4y + 8x^2y^3z - 12x^3yz^2\).

**Ans:** Take out THE COMMON TERMS

\[
6x^4y + 8x^2y^3z - 12x^3yz^2 = 2 \times 3 \times x^2 \times y \times y \times y \times x \times y \times z - 2 \times 6 \times x^2 \times x \times y \times x \times z
\]

\[
2(3x^2y + 4y^2x - 6x^2yz)
\]

\[
2x^2y(3x^2 + 4y^2z - 6xz^2)
\]

2 is the biggest number that is common in 6, 8, 12

Highest powers of x and y that is common in all three terms

z wasn't in ALL terms so it can't come out as a common factor

Example: Simplify \(-4x + 2y - 2 - y + x\).

**Ans:** Combine LIKE TERMS
Example: Simplify \(8w + 6k - 5w - 12k^2 + 8\).

Ans: Combine LIKE TERMS

\[
8w + 6k - 5w - 12k^2 + 8 = 3w + 6k - 12k^2 + 8
\]

Example: Expand \(2ab(4a - 3b^2)\).

Ans: Multiply THE TERMS

\[
2ab(4a - 3b^2) = 8a^2b - 6ab^3
\]

Example: Expand \((2f + 3)(5f - 4)\).

Ans: Multiply THE TERMS

\[
(2f + 3)(5f - 4) = 10f^2 - 8f + 15f - 12 = 10f^2 + 7f - 12
\]

Example: Expand \((1 - 2x)^2\).

Ans: Multiply THE TERMS
Example: Expand \((3x - 2)^2\).

Ans: Multiply THE TERMS

\[(3x - 2)(3x - 2) = 9x^2 - 6x - 6x + 4 = 9x^2 - 12x + 4\]

Example: Factorise \(12q^2r^3 - 24q^2r + 30q^3r^4\).

Ans: Take out THE COMMON TERMS

\[12q^2r^3 - 24q^2r + 30q^3r^4 \quad \quad 6\] is the biggest number that is common in 6, 8, 12

Highest powers of \(q\) and \(r\) that is common in all three terms

\[6q^2 r(2r^2 - 4 + 5q r^3)\]

Example: Factorise \(6x^3y^2 - 3x^2y + 12xy^3z\).

Ans: Take out THE COMMON TERMS

\[6x^3y^2 - 3x^2y + 12xy^3z\]
3 is the biggest number
that is common in 6, 8, 12

\[ 3 \text{xy}(2x^2y - x + 4y^2z) \]

Highest powers of \( x \) and \( y \) that is common in all
three terms

\( z \) wasn’t in ALL terms so
it can’t come out as a
common factor

Example: Simplify \( 3(3x - 8) \).
Ans: \( 3(3x - 8) = 9x - 24 \)

Example: Simplify \( 3(3a - 8b) \)
Ans: \( 3(3a - 8b) = 9a - 24b \)

Example: Simplify \( 3a(3a - 8b) \)
Ans: \( 3a(3a - 8b) = 9a^2 - 24ab \)

Example: Simplify \( 3a^2(3a - 8b) \)
Ans: \( 3a^2(3a - 8b) = 9a^3 - 24a^2b \)

Example: Simplify \( \frac{3}{4}ab^2 + \frac{1}{2}a^2b^3 - \frac{5}{7}abc \).
Ans: \( \frac{3}{4}ab^2 + \frac{1}{2}a^2b^3 - \frac{5}{7}abc = ab \left( \frac{3}{4}b + \frac{1}{2}ab^2 - \frac{5}{7}c \right) \)

Example: Simplify \( \frac{3}{4}a^2b + \frac{1}{2}ab^3 - \frac{3}{8}ab \).
Ans: \( \frac{3}{4}a^2b + \frac{1}{2}ab^3 - \frac{3}{8}ab = \frac{1}{2}ab \left( \frac{3}{2}a + \frac{1}{2}b^2 - \frac{3}{4} \right) \) Note: \( \frac{3}{4} = \frac{1}{2} \times \frac{3}{2}, \quad \frac{3}{8} = \frac{1}{2} \times \frac{3}{4} \)

Example: Simplify \( 3(3x - 8) + 4(5 - 2x) - 5(2x - 7) \).
Ans: \( 3(3x - 8) + 4(5 - 2x) - 5(2x - 7) = 9x - 24 + 20 - 8x - 10x + 35 = -9x + 31 \)

Example: Simplify \( -3(-3x - 8) + 4(-5 - 2x) - 5(2x - 7) \).
Ans: \( -3(-3x - 8) + 4(-5 - 2x) - 5(2x - 7) = 9x + 24 - 20 - 8x - 10x + 35 = -9x + 39 \)
Example: Simplify \( \frac{x + 2}{3} + \frac{x - 3}{2} - \frac{2x + 3}{5} \).

Ans:

\[
\frac{x + 2}{3} + \frac{x - 3}{2} - \frac{2x + 3}{5} = \frac{10(x + 2) + 15(x - 3) - 6(2x + 3)}{30} = \frac{10x + 20 + 15x - 45 - 12x - 18}{30} = \frac{13x - 43}{30}
\]

Example: Simplify \( \frac{x + 2}{3} \times \frac{x - 3}{2} = \frac{(x + 2)(x - 3)}{3 \times 2} = \frac{x^2 - 3x + 2x - 6}{6} = \frac{x^2 - x - 6}{6} \)

Example: Simplify \( \frac{x + 2}{3} \times \frac{x - 3}{2} = \frac{x + 2}{3} \times \frac{2}{x - 3} = \frac{2(x + 2)}{3(x - 3)} = \frac{2x + 4}{3x - 9} \)
LESSON 7

Solving the Equations

\[\begin{align*}
\text{All unknown to this side} & = \text{All known to this side} \\
\end{align*}\]

Example: Solve \(2x - 8 = 24\).

Ans: \[
\begin{align*}
2x - 8 &= 24 \\
2x &= 24 + 8 \\
x &= \frac{32}{2} = 16 \\
\end{align*}
\]

Check: \(2x - 8 = 24\) (not needed in the exam)
\[
\begin{align*}
2 \times 16 - 8 &= 24 \\
32 - 8 &= 24 \\
24 &= 24 \quad \text{O.K}
\end{align*}
\]

Example: Solve \(2a + 4 = 6a - 4\).

Ans: \[
\begin{align*}
2a + 4 &= 6a - 4 \\
2a - 6a &= -4 - 4 \\
-4a &= -8 \\
a &= \frac{-8}{-4} = 2 \\
\end{align*}
\]

Example: Solve \(2(x - 1) = 3(2x - 5) + 1\).

Ans: \[
\begin{align*}
2(x - 1) &= 3(2x - 5) + 1 \\
2x - 2 &= 6x - 15 + 1 \\
2x - 6x &= -15 + 1 + 2 \\
-4x &= -12 \\
x &= \frac{-12}{-4} = 3 \\
\end{align*}
\]
Example: Solve \( \frac{x + 1}{4} = \frac{4x - 7}{5} \).  
\[ \text{Note: If } \frac{a}{b} = \frac{c}{d} \Rightarrow ad = bc \]

\[
\begin{align*}
\text{Ans: } \quad & \frac{x + 1}{4} = \frac{4x - 7}{5} \\
& 4(4x - 7) = 5(x + 1) \\
& 16x - 28 = 5x + 5 \\
& 16x - 5x = 28 + 5 \\
& 11x = 33 \Rightarrow x = \frac{33}{11} = 3
\end{align*}
\]

Example: Solve \( \frac{x + 1}{4} - \frac{4x - 7}{5} = 0 \).

\[
\begin{align*}
\text{Ans: } \quad & \frac{x + 1}{4} = \frac{4x - 7}{5} \\
& 4(4x - 7) = 5(x + 1) \\
& 16x - 28 = 5x + 5 \\
& 16x - 5x = 28 + 5 \\
& 11x = 33 \\
& x = \frac{33}{11} = 3
\end{align*}
\]

Example: Solve \( \frac{1}{4}x + \frac{1}{4} = \frac{4}{5}x - \frac{7}{5} \).

\[
\begin{align*}
\text{Ans: } \quad & \frac{1}{4}x - \frac{4}{5}x = -\frac{7}{5} - \frac{1}{4} \\
& \frac{5 - 16}{20} x = -\frac{28 - 5}{20} \\
& \frac{-11}{20} x = -\frac{33}{20} \\
& -11x = -33 \\
& x = 3
\end{align*}
\]

Example: Solve \( \frac{6}{n + 1} = -3 \).

\[
\begin{align*}
\text{Ans: } \quad & \frac{6}{n + 1} = -3 \\
& \frac{6}{n + 1} = -3 \\
& \frac{-3}{1} = -3 \\
& -3(n + 1) = 6
\end{align*}
\]
\[-3n - 3 = 6\]
\[-3n = 6 + 3\]
\[-3n = 9\]
\[n = \frac{9}{-3} = -3\]

**Example:** Solve \(\frac{10 - 4}{m} = 7\).

**Ans:** \(\frac{4}{m} = 7 - 10\)

\[-\frac{4}{m} = -3\]

\[-\frac{4}{m} = \frac{3}{1}\]
\[-\frac{4}{m} = \frac{-3}{1}\]

\[-3m = -4\]

\[3m = 4\]
\[m = \frac{4}{3}\]

**Example:** If I add 7 to my Mum’s age and then divide by 5. I get the same answer as if I subtract 2 from my Mum’s age and then divide by 4. Work out my Mum’s age.

**Ans:** Let Mum’s age = \(x\)

\[\frac{x + 7}{5} = \frac{x - 2}{4}\]

\[4(x + 7) = 5(x - 2)\]
\[4x + 28 = 5x - 10\]
\[4x - 5x = -10 - 28\]
\[-x = -38\]
\[x = 38\]

**Check:**

\[\frac{38 + 7}{5} = \frac{38 - 2}{4}\]
\[\frac{45}{5} = \frac{36}{4}\]
\[9 = 9 \quad \text{O.K}\]
Example: Solve \( 3(x + 1) = 2 + 4(2 - x) \).

\[
\begin{align*}
\text{Ans:} & \quad 3x + 3 = 2 + 8 - 4x \\
& \quad 3x + 4x = 2 + 8 - 3 \\
& \quad 7x = 7 \\
& \quad x = \frac{7}{7} = 1
\end{align*}
\]

Example: Solve \( \frac{6}{x+3} = \frac{9}{5+2x} \).

\[
\begin{align*}
9(x + 3) &= 6(5 + 2x) \\
9x + 27 &= 30 + 12x \\
9x - 12x &= 30 - 27 \\
-3x &= 3 \\
x &= -\frac{3}{3} = -1
\end{align*}
\]

Example:

\[
\begin{align*}
\frac{3}{x+2} + \frac{2}{x-2} &= 0 \\
\frac{3}{x+2} &= -\frac{2}{x-2} \\
\frac{3}{x+2} &= \frac{-2}{(x-2)} \\
3(x-2) &= -2(x+2) \\
3x - 6 &= -2x - 4 \\
3x + 2x &= -4 + 6 \\
5x &= 2 \\
x &= \frac{2}{5} = 0.4
\end{align*}
\]

OR

\[
\begin{align*}
\frac{3}{x+2} + \frac{2}{x-2} &= 0 \\
\frac{3}{x+2} &= -\frac{2}{x-2} \\
\frac{3}{x+2} &= \frac{-2}{(x-2)} \\
-3(x-2) &= 2(x+2) \\
-3x + 6 &= 2x + 4 \\
-3x - 2x &= 4 - 6 \\
-5x &= -2 \\
x &= \frac{-2}{-5} = 0.4
\end{align*}
\]
Example: \[ \frac{x+2}{3} - \frac{x-3}{2} = 1 \]

Ans: Multiply each side by 6

\[
6 \times \left( \frac{x+2}{3} - \frac{x-3}{2} \right) = 6 \times 1 \\
2(x+2) - 3(x-3) = 6 \\
2x + 4 - 3x + 9 = 6 \\
-x = 6 - 13 \\
x = -7 \\
x = 7
\]

**LESSON 8**

**Turning words into Algebra**

**Example:** A number is doubled, then 3 is added to the total, and the result is 15. What was the original number?

**Ans:** Let the original number = \( x \)

\[
2x + 3 = 15 \\
2x = 15 - 3 \\
2x = 12 \\
x = \frac{12}{2} = 6
\]

**Example:** A group of 4 workers were paid £15 per hour plus a tip of £6. They shared the takings and each got £9. How many hours did they work?

**Ans:** Let the number of hours = \( x \)

\[
\frac{15x + 6}{4} = 9 \\
15x + 6 = 36 \\
15x = 36 - 6 \\
15x = 30 \\
x = \frac{30}{15} = 2
\]

**Example:** If I add 7 to my Mum’s age and then divide by 5, I get the same answer as if I subtract 2 from my Mum’s age and then divide by 4. Work out my Mum’s age.
Ans: Let Mum’s age = x

\[
\frac{x + 7}{5} = \frac{x - 2}{4}
\]

\[4(x + 7) = 5(x - 2)\]

\[4x + 28 = 5x - 10\]

\[4x - 5x = -10 - 28\]

\[-x = -38\]

\[x = 38\]

Check:

\[\frac{38 + 7}{5} = \frac{38 - 2}{4}\]

\[\frac{45}{5} = \frac{36}{4}\]

\[9 = 9 \text{   O.K}\]

Example: My brother is twice as old as I am and the sum of our ages is 45. How old is my brother?

Ans: Let my brother age = x

\[
\frac{x}{2} + x = 45
\]

\[\frac{x + 2x}{2} = 45\]

\[3x = 90\]

\[x = 30\]

Example: I think of a number, treble it, add seven and the answer is 19. Form an equation using this information and find that number?

Ans: Let the number = x

\[3x + 7 = 19\]

\[3x = 19 - 7\]

\[3x = 12\]

\[x = 4\]

Example: I buy a pizza and cut it into three pieces. When I weigh the pieces, I find that one piece is 8 g lighter than the largest piece and 5 g heavier than the smallest piece. If the whole pizza weights 360 g, how much does the smallest piece weight?

Ans: Let the weight of the smallest piece = x

The weight of the other pieces are = x+5 and x+5+8

\[x + (x+5) + (x+5+8) = 360\]

\[x + x + x + 5 + 5 + 8 = 360\]
LESSON 9

Trigonometry

\[ \sin \theta = \frac{\text{Opposite Side}}{\text{Hypotenuse}} = \frac{BC}{AB} \]

\[ \cos \theta = \frac{\text{Adjacent Side}}{\text{Hypotenuse}} = \frac{AC}{AB} \]

\[ \tan \theta = \frac{\text{Opposite Side}}{\text{Adjacent Side}} = \frac{BC}{AC} \]

Decimal of a Degree

Example: Convert \( 28^\circ 42' \) into degrees and decimal of a degree.

Ans: \( 28^\circ 42' = 28 \frac{42}{60} = 28 \frac{7}{10} = 28.7^\circ \)
Example: Convert $28.7^\circ$ into degrees and minutes.

Ans: $28.7^\circ = 28^\circ (60 \times 0.7)' = 28^\circ 42'$

**The Sine of an Angle**

Example: Find the length of the side BC.

[Diagram of a triangle with side lengths labeled: BC = 12 cm, angle C = 42°]

Ans: \[
\sin 42^\circ = \frac{BC}{AC} = \frac{12}{AC} \]

\[
\sin 42^\circ = \frac{12}{AC}
\]

$BC = 12 \times \sin 42^\circ = 12 \times 0.6691 = 8.03 cm$

Example: Find the length of the side AC.

[Diagram of a triangle with side lengths labeled: AB = 5 cm, angle C = 17.8°]

Ans: \[
\sin 17.8^\circ = \frac{AB}{AC} \]

\[
\sin 17.8^\circ = \frac{5}{AC}
\]

\[
AC = \frac{5}{\sin 17.8^\circ} = \frac{5}{0.3057} = 16.4 cm\]
Example: Find the angles $A$ and $B$.

![Diagram of a triangle with sides 8 cm, 11 cm, and unknown angles $A$ and $B$.]

Ans: \[
\sin A = \frac{BC}{AB} \\
\sin A = \frac{8}{11} = 0.7273 \\
\angle A = \sin^{-1}(0.7273) = 46.7^\circ \\
\angle B = 90^\circ - 46.7^\circ = 43.3^\circ
\]

Example: Find the length of the sides marked $x$.

![Diagram of three triangles with sides 20 cm, 35°, and unknown side $x$; 18 cm, 42°, and unknown side $x$; 8 cm, 26°, and unknown side $x$.]

Ans: \[
\sin 35^\circ = \frac{x}{20} \\
x = 20 \times \sin 35^\circ = 20 \times 0.5735764 = 11.47153 \text{ cm}
\]

Ans: \[
\sin 42^\circ = \frac{x}{18} \\
x = 18 \times \sin 42^\circ = 18 \times 0.6691306 = 12.044351 \text{ cm}
\]

Ans: \[
\sin 26^\circ = \frac{8}{x} \\
x = \frac{8}{\sin 26^\circ} = \frac{8}{0.4383711} = 18.249376 \text{ cm}
\]
The Cosine of an Angle

Example: Find the length of the side AB.

\[ \cos 42^0 = \frac{AB}{AC} \]
\[ \cos 42^0 = \frac{AB}{12} \]
\[ AB = 12 \times \cos 42^0 = 12 \times 0.743145 = 8.918 \text{ cm} \]

Example: Find the length of the side AC.

\[ \cos 17.8^0 = \frac{BC}{AC} \]
\[ \cos 17.8^0 = \frac{5}{AC} \]
\[ AC = \frac{5}{\cos 17.8^0} = \frac{5}{0.95213} = 5.2514 \text{ cm} \]
Example: Find the angles A and B.

![Triangle with sides 8 cm, 11 cm, and an unknown side]

**Ans:**
\[
\cos B = \frac{BC}{AB} = \frac{8}{11} = 0.7273
\]
\[
\angle B = \cos^{-1}(0.7273) = 43.34^\circ
\]
\[
\angle A = 90^\circ - 43.34^\circ = 46.66^\circ
\]

Example: Find the length of the sides marked x.

![Triangle with sides 18 cm, 42 cm, and an unknown side]

**Ans:**
\[
\cos 35^\circ = \frac{x}{20}
\]
\[
x = 20 \times \cos 35^\circ = 20 \times 0.819152 = 16.383041 \text{ cm}
\]

**Ans:**
\[
\cos 42^\circ = \frac{x}{18}
\]
\[
x = 18 \times \cos 42^\circ = 18 \times 0.7431448 = 13.376607 \text{ cm}
\]

**Ans:**
\[
\cos 26^\circ = \frac{8}{x}
\]
\[
x = \frac{8}{\cos 26^\circ} = \frac{8}{0.898794} = 8.9008155 \text{ cm}
\]
The Tangent of an Angle

Example: Find the length of the side $AB$.

\[ \tan 42^\circ = \frac{BC}{AB} \]
\[ \tan 42^\circ = \frac{12}{AB} \]
\[ AB = \frac{12}{\tan 42^\circ} = \frac{12}{0.9} = 13.33 \text{ cm} \]

Example: Find the length of the side $AB$.

\[ \tan 17.8^\circ = \frac{AB}{BC} \]
\[ \tan 17.8^\circ = \frac{AB}{5} \]
\[ AB = 5 \times \tan 17.8^\circ = 5 \times 0.3211 = 1.605 \text{ cm} \]
Example: Find the angles $A$ and $B$.

![Diagram of a triangle with sides labeled 8 cm, 11 cm, and unknown lengths $A$, $B$, and $C$.]

 Ans: $\tan A = \frac{BC}{AC}$

$\tan A = \frac{11}{8} = 1.375$

$\angle A = \tan^{-1}(1.375) = 53.973^\circ$

$\angle B = 90^\circ - 53.973^\circ = 36.027^\circ$

Example: Find the length of the sides marked $x$.

![Diagram of two triangles with sides labeled 18 cm, 35 cm, and unknown lengths $x$.]

 Ans: $\tan 35^\circ = \frac{x}{20}$

$x = 20 \times \tan 35^\circ = 20 \times 0.7002075 = 14.004151 \text{ cm}$

 Ans: $\tan 42^\circ = \frac{18}{x}$

$x = \frac{18}{\tan 42^\circ} = \frac{18}{0.900404} = 19.991025 \text{ cm}$
Ans: \( \tan 26^\circ = \frac{8}{x} \)

\[
x = \frac{8}{\tan 26^\circ} = \frac{8}{0.4877325} = 16.402431 \text{ cm}
\]

**Example:** If \( \sin \theta = \frac{3}{5} \), find \( \cos \theta \) and \( \tan \theta \).

**Ans:**

\[
\cos \theta = \frac{4}{5} \quad \text{and} \quad \tan \theta = \frac{3}{4}
\]

**Example:** If \( \cos \theta = \frac{4}{5} \), find \( \sin \theta \) and \( \tan \theta \).

**Ans:**

\[
\sin \theta = \frac{3}{5} \quad \text{and} \quad \tan \theta = \frac{3}{4}
\]

**Example:** If \( \tan \theta = \frac{3}{4} \), find \( \sin \theta \) and \( \cos \theta \).

**Ans:**

\[
\sin \theta = \frac{3}{5} \quad \text{and} \quad \cos \theta = \frac{4}{5}
\]
Example: In the triangle ABC, $\angle B = 90^\circ$, $\angle C = 26^\circ 21'$ and $b = 13.4\text{ cm}$.
Calculate the length of the side $c$ of the triangle.

\[
\sin C = \frac{c}{b} \Rightarrow c = b \sin C = 13.4 \times \sin 26.35^\circ = 5.9476349 \approx 5.95 \text{ (2.d.p)}
\]

Example: In the triangle ABC, $\angle C = 90^\circ$, $\angle A = 69.3^\circ$ and $a = 3.4\text{ cm}$.
Calculate the length of the side $c$ of the triangle.

\[
c = \frac{a}{\sin A} = \frac{3.4}{\sin 69.3^\circ} = 3.6346375 \approx 3.63 \text{ (2.d.p)}
\]

Example: An equilateral triangle has a vertical height of $20\text{ cm}$.
Calculate the length of the equal sides.

\[
\sin 60^\circ = \frac{h}{BC} \Rightarrow BC = \frac{h}{\sin 60^\circ} = \frac{20}{0.8660254} = 23.094011 \approx 23.09 \text{ (2.d.p)}
\]
Example: Calculate the length of the equal sides of an isosceles triangle whose altitude is $15\text{ cm}$ and whose equal angles are $48^0 36'$. 

**Ans:** $48^0 36' = 48 \frac{36}{60} = 48 \frac{6}{10} = 48.6^0$

![Diagram of an isosceles triangle with altitude](image)

$$\sin 48.6^0 = \frac{h}{BC}$$

$$BC = \frac{h}{\sin 48.6^0} = \frac{15}{0.750111} = 19.997039 \approx 20.00\text{ (2.d.p)}$$

Example: The equal sides of an isosceles triangle are each $12\text{ cm}$ long. If the altitude of the triangle is $8\text{ cm}$, find the size of the three angles of the triangle.

**Ans:**

![Diagram of an isosceles triangle with altitude](image)

$$\sin C = \frac{h}{BC}$$

$$\sin C = \frac{8}{12} = 0.666666$$

$$C = \sin^{-1}(0.666666) = 41.81^0$$

$$\angle A = \angle C = 41.81^0$$

$$\angle B = 180^0 - (41.81^0 + 41.81^0) = 96.38^0$$
Example: In the triangle ABC, \(\angle A = 90^0\), \(b=10.8\,cm\) and \(a=12.3\,cm\) are each \(b=10.8\,cm\) long. Find the size of the angles \(B\) and \(C\).

Ans:

\[
\sin B = \frac{b}{a}
\]

\[
\sin B = \frac{10.8}{12.3} = 0.8780487
\]

\[
\angle B = \sin^{-1}(0.8780487) = 61.407877^0 \approx 61.41^0
\]

\[
\angle C = 180^0 - (90^0 + 61.41^0) = 28.59^0
\]

Example: Calculate the vertical height of an isosceles triangle with an apex angle of \(42^0\) and equal sides \(8\,cm\) long.

Ans:

\[
\angle CBD = \frac{42}{2} = 21^0
\]

\[
\cos 21^0 = \frac{h}{BC} = \frac{h}{8} \Rightarrow h = 8 \times \cos 21^0 = 7.4686434 \approx 7.47\,cm\,(2\,d.p)
\]

Example: The base of an isosceles triangle is \(12\,cm\) long and the equal sides are \(15\,cm\) long. Find the three angles of the triangle.

Ans:
Example: In the triangle ABC, $\angle C = 90^0$, $\angle B = 35^0 48'$ and $a = 9\text{ cm}$.

Find the length of the side $c$.

**Ans:**

$$\frac{DC}{BC} = \frac{6}{15} = 0.4 \Rightarrow \angle C = \cos^{-1}(0.4) = 66.42^0$$

$$\angle A = \angle C = 66.42^0 \Rightarrow \angle B = 180 - (66.42 + 66.42) = 47.16^0$$

Example: In Fig.1, find $\angle BAC$ and the length of BD

**Ans:**

$$\cos B = \frac{a}{c}$$

$$c = \frac{a}{\cos B} = \frac{9}{\cos 35.8^0} = \frac{9}{0.8110638} = 11.096537 \approx 11.1\text{ cm}$$

$$\cos A = \frac{4}{8} = 0.5 \Rightarrow A = \cos^{-1}(0.5) = 60^0$$

$$BD = \sqrt{8^2 - 4^2} = \sqrt{64 - 16} = \sqrt{48} = 6.9282032 \approx 6.93 \text{ cm} (2\text{ d.p})$$
Example: The base of an isosceles triangle is 12 cm long and the equal angles are each 54°. Calculate the altitude of the triangle.

[Diagram of isosceles triangle]

Ans:

\[ AD = \frac{AC}{2} = \frac{12}{2} = 6 \text{ cm} \]

\[ \tan A = \frac{h}{AD} = \frac{h}{6} \Rightarrow h = 6 \times \tan 54^\circ = 6 \times 1.3763819 = 8.2582915 \approx 8.26 \text{ cm} \ (2 \text{ d.p}) \]

Example: Calculate the distance marked \( a \) in Fig. 2.

[Diagram of Fig. 2]

Ans:

\[ \tan 39^\circ = \frac{a}{7.2 - 4.7} = \frac{a}{2.5} \Rightarrow a = 2.5 \times \tan 39^\circ = 2.5 \times 0.809784 = 2.0244601 \approx 2.02 \text{ cm} \ (2 \text{ d.p}) \]
LESSON 10

Angles of Elevation and Depression

If you look upwards at an object, the angle formed between the horizontal and your line of sight is called the angle of elevation (See Fig.1).

Example: A boy 1.5 m tall is 25 m away from a tower as shown in Fig.2. The angle of elevation of the tower is 18°. How high is the tower?

\[ \tan 18^0 = \frac{TR}{SR} \Rightarrow TR = SR \times \tan 18^0 = 25 \times 0.3249197 = 8.1229924 \approx 8.12 \text{ m} (2 \text{ d.p}) \]

\[ UT = 8.12 \text{ m} + 1.5 \text{ m} = 9.62 \text{ m} \]
If you look downwards at an object, the angle formed between the horizontal and your line of sight is called the **angle of depression** (See Fig.3).

![Fig.3](image-url)

**Example:** A boy standing on top of a cliff 50 m high is in line with two buoys whose angles of depression are 18° and 20°. Calculate the distance between the buoys. The problem is illustrated in Fig.4, where the buoys are C and D and the observer is A.

![Fig.4](image-url)

**Ans:** In the triangle ABC, \( \angle BAC = 90° - 20° = 70° \)

\[
\tan \angle BAC = \tan 70° = \frac{BC}{AB} \Rightarrow BC = AB \times \tan 70° = 50 \times 2.74774774 = 137.37387 \approx 137.37 \text{ m (2.d.p)}
\]

In the triangle ABD, \( \angle BAD = 90° - 18° = 72° \)

\[
\tan \angle BAD = \tan 72° = \frac{BD}{AB} \Rightarrow BD = AB \times \tan 72° = 50 \times 3.0776835 = 153.88418 \approx 153.88 \text{ m (2.d.p)}
\]

\[
CD = BD - BC = 153.88 - 137.37 = 16.51 \text{ m (2.d.p)}.
\]

Therefore the distance between the buoys is 16.51 m.
**Altitude of the Sun**

The altitude of the sun is the angle of elevation of the sun (Fig.5).

![Diagram of altitude of the sun](image)

**Example:** A flagpole is 11 m tall and it stands vertically. Calculate the length of shadow that it will cast when the altitude of the sun is 43°.

![Diagram of flagpole and shadow](image)

**Ans:**

\[
\tan 43^\circ = \frac{BC}{AB} \Rightarrow AB = \frac{BC}{\tan 43^\circ} = \frac{11}{0.932515} = 11.796056 \approx 11.80 \text{m (2.d.p)}
\]

Therefore the length of the shadow cast by the flagpole is 11.80 m.
Area of a triangle

$A = \frac{1}{2}bh$

$A = \frac{1}{2}bc\sin A$

$A = \frac{1}{2}ac\sin B$

$A = \frac{1}{2}abs\sin C$

$A = \sqrt{s(s-a)(s-b)(s-c)}$

$s = \frac{a+b+c}{2}$

Bearings

Note: The usual way of stating bearing is to measure the angle from North in a clockwise direction.

Example: P is a point due east of a harbour H and Q is a point on the coast 10 km due south of H. If the distance PQ is 14 km, find the bearing of Q from P.

\[
\sin \angle HPQ = \frac{HQ}{PQ} = \frac{10}{14} = 0.7143 \Rightarrow \angle HPQ = 45.6^0
\]

Bearing of Q from P $270^0 - 45.6^0 = 224.4^0$
Example: The bearing of a point B from a point A is $65^\circ$. What is the bearing of A from B?

\[ \text{Bearing of A from B} = 270^\circ - 25^\circ = 245^\circ \]

Example: Bearing of a point C from a point B is $225^\circ$. Find:
(a) the bearing of A from B.
(b) the size of the angle ABC.

Ans:

\[ \angle ABC = 360^\circ - (225^\circ + 60^\circ) = 75^\circ \]

\[ \text{Bearing of A from B} = 225^\circ + 75^\circ = 300^\circ \]

Example: Find the bearing of X from Y.
Example: The distance from town A to town B and C are 160 km and 340 km respectively. The bearing of B from A is 346° and the bearing of C from A is 35°. Find:
(a) the distance between B and C
(b) the bearing of C from B.

Note: To find the bearing of a point B from a point A,
1. Join A and B.
2. Draw in the North line at A.
3. Find the angle between North and AB, measured clockwise.
4. Record the angle, in degrees, as a three-digit figure.

The distance from town A to town B and C are 160 km and 340 km respectively. The bearing of B from A is 346° and the bearing of C from A is 35°. Find:
(a) the distance between B and C
(b) the bearing of C from B.

Note: To find the bearing of a point A from a point B,
1. Join A and B.
2. Draw in the North line at B.
3. Find the angle between North and AB, measured clockwise.
4. Record the angle, in degrees, as a three-digit figure.

Example: In the diagram, the bearing of A from B is 250°. Find the bearing of B from A.

\[
\begin{array}{c}
\text{N} \\
\downarrow \\
A \quad 250° \\
\downarrow \\
\text{B} \\
\text{N}
\end{array}
\quad \quad \quad
\begin{array}{c}
\text{N} \\
\downarrow \\
A \\
\downarrow \\
B \quad 250° \\
\downarrow \\
\text{N}
\end{array}
\]

Bearing of A from B is 250°
Bearing of B from A is 70°

Example: The bearing of Q from P is 192°. What is the bearing of P from Q.

\[
\begin{array}{c}
\text{N} \\
\downarrow \\
Q \quad 192° \\
\downarrow \\
P \\
\text{N}
\end{array}
\quad \quad \quad
\begin{array}{c}
\text{N} \\
\downarrow \\
P \quad 192° \\
\downarrow \\
Q \quad 168° \\
\downarrow \\
\text{N}
\end{array}
\]

Bearing of Q from P is 192°
Bearing of P from Q is 12°
LESSON 11

Transformations

There are four basic transformations.

1. Translation

A translation moves every point of an object in the same direction, through the same distance.
To change position from p to p', the point is moved 6 units horizontally and 3 units vertically. This translation is represented by the vector \[ \begin{pmatrix} 6 \\ 3 \end{pmatrix} \]. (See Fig.1).

Example: Draw the rectangle ABCD with vertices A(3,1), B(5,1), C(5,4) and D(3,4). The rectangle is transformed by the translation \[ \begin{pmatrix} -3 \\ 2 \end{pmatrix} \] into rectangle A'B'C'D'. Draw rectangle A'B'C'D'.

Solution (See Fig.2)

Position of A' = Position of A + Translation = \[ \begin{pmatrix} 3 \\ 1 \end{pmatrix} + \begin{pmatrix} -3 \\ 2 \end{pmatrix} = \begin{pmatrix} 0 \\ 3 \end{pmatrix} \].
Position of $B'$ = Position of $B$ + Translation = \[
\begin{pmatrix} 5 \\ 1 \\ 2 \end{pmatrix} + \begin{pmatrix} -3 \\ 2 \end{pmatrix} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}
\]

Position of $C'$ = Position of $C$ + Translation = \[
\begin{pmatrix} 5 \\ 4 \\ 2 \end{pmatrix} + \begin{pmatrix} -3 \\ 2 \end{pmatrix} = \begin{pmatrix} 2 \\ 6 \end{pmatrix}
\]

Position of $D'$ = Position of $D$ + Translation = \[
\begin{pmatrix} 3 \\ 4 \\ 2 \end{pmatrix} + \begin{pmatrix} -3 \\ 2 \end{pmatrix} = \begin{pmatrix} 0 \\ 6 \end{pmatrix}
\]

Example No: Write down, as vectors, the translations which move triangle ABC to each of the positions a, b, c, d and e.
Ans: (See Fig.3)

(a) \[ \begin{pmatrix} 0 \\ -6 \end{pmatrix} \]  
(b) \[ \begin{pmatrix} 9 \\ -5 \end{pmatrix} \]  
(c) \[ \begin{pmatrix} 9 \\ 0 \end{pmatrix} \]  
(d) \[ \begin{pmatrix} 5 \\ 3 \end{pmatrix} \]  
(e) \[ \begin{pmatrix} -5 \\ 4 \end{pmatrix} \]  

Example No: Triangle ABC is transformed into triangle A'B'C' by the translation \[ \begin{pmatrix} 4 \\ 3 \end{pmatrix} \].

A is (2,1), B is (3,5), and C is (−1,−2). Find the coordinates of A’, B’ and C’.

Solution

Position of A’ = Position of A + Translation = \[ \begin{pmatrix} 2 \\ 1 \end{pmatrix} + \begin{pmatrix} 4 \\ 3 \end{pmatrix} = \begin{pmatrix} 6 \\ 4 \end{pmatrix} \].

Position of B’ = Position of B + Translation = \[ \begin{pmatrix} 3 \\ 5 \end{pmatrix} + \begin{pmatrix} 4 \\ 3 \end{pmatrix} = \begin{pmatrix} 7 \\ 8 \end{pmatrix} \].

Position of C’ = Position of C + Translation = \[ \begin{pmatrix} -1 \\ -2 \end{pmatrix} + \begin{pmatrix} 4 \\ 3 \end{pmatrix} = \begin{pmatrix} 3 \\ 1 \end{pmatrix} \].

2. Reflection

A reflection moves an object so that it becomes a mirror image of itself. (See Fig.4).
Example: Reflect the triangle in the mirror line (see Fig.5)

Solution: Reflect each corner of the triangle in the mirror line and joins the three images (see Fig.6)
Example: The diagram shows a triangle S and its image, triangle T, after a reflection (see Fig.7). Draw the mirror line of the reflection.

Solution: Join each corner of triangle S to its image on triangle T. The mirror line passes through the midpoints (marked with crosses) of these lines (see Fig.8).

Example: Describe the transformation which maps triangle A onto triangle B.

Solution: The transformation is a reflection. The mirror line has the equation $x = 1$. The transformation is a reflection in the line $x = 1$.

Example: Describe fully the transformation which maps triangle P onto triangle Q.

Solution: The transformation is a reflection in the line $y = x$. 

64
LESSON 12

Upper and Lower Bounds

If you think of the range of possible values, the bottom end of the range is called the **lower bound** (lower limit), and the top end of the range is called the **upper bound** (upper limit).

Example:  A work surface is \(3 \text{m}\) long to the nearest \(\text{cm}\). What are the limits of its actual lengths?

Ans: Because, it is to the nearest \(\text{cm}\), then it would be by \(0.5 \text{ cm}\) each side. Hence: \(2.995 \leq 3 \text{m} < 3.005\).

Example: Give the upper and lower bounds for the following amounts:

(a) \(\£12\) (to the nearest \(\£\))

Ans: \(11.50 \leq \£12 < 12.49\)

(b) \(\£60\) (to the nearest \(\£10\))

Ans: \(55.00 \leq \£60 < 64.99\)

(c) \(\£750\) (to the nearest \(\£50\))

Ans: \(725.00 \leq \£750 < 774.99\)

(d) \(\£8.90\) (to the nearest \(10\text{p}\))

Ans: \(8.85 \leq \£8.90 < 8.94\)

Example: Give the upper and lower bounds for the following amounts:

(a) \(638 \text{ kg}\) (to the nearest \(\text{kg}\))

Ans: \(637.5 \leq 638 \text{ kg} < 638.5\)

(b) \(1700 \text{ g}\) (to the nearest \(100 \text{ g}\))

Ans: Subtract and Add \(\frac{100}{2} = 50 \text{ g}\) to both: \(1650 \leq 1700 \text{ g} < 1750\)

(c) \(10 \text{ m}\) (to the nearest \(\text{m}\))

Ans: Subtract and Add \(\frac{1}{2} = 0.5 \text{ m}\) to both: \(9.5 \leq 10 \text{ m} < 10.5\)

(d) \(495 \text{ cm}\) (to the nearest \(5 \text{ cm}\))

Ans: Subtract and Add \(\frac{5}{2} = 2.5 \text{ cm}\) to both: \(492.5 \leq 495 \text{ cm} < 497.5\)
If the degree of accuracy is not given in a question it may be implied by the way the number is written. If a measurement is given as 4.6 m the possible range of values is $4.55 \leq 4.6 < 4.65$. If a measurement is given as 4.60 m the possible range of values is $4.595 \leq 4.60 < 4.605$.

**Example:** Give the upper and lower bounds for the following:

(a) A room is 3.7 m long.  
**Ans:** $3.65 \leq 638 \text{ kg} < 3.75$

(b) A newspaper report indicates that the attendance at a football match was 11000.  
**Ans:** $10500 \leq 11000 < 11499$.

(c) An object is weighed on two different sets of scales, one of which is more accurate than the other. The scale shows the following amounts:  
(i) 2.61 kg  
(ii) 2.610 kg.  
**Ans:** (i) $2.605 \leq 2.61 \text{ kg} < 2.615$  
(ii) $2.6095 \leq 2.610 \text{ kg} < 2.6105$

Note: Find the upper bound for the perimeter of a rectangular building plot which has sides of 40 m and 18 m measured to the nearest metre.  
**Ans:** The upper bounds of the plot’s measurements are 40.5 m and 18.5 m. The upper bound of the perimeter is therefore $2(40.5+18.5)=118$ m.

**Example:** Calculate the lower bound length for the perimeter of the building plot.  
**Ans:** The lower bounds of the plot’s measurements are 39.5 m and 17.5 m. The lower bound of the perimeter is therefore $2(39.5+17.5)=114$ m.

**Example:** Calculate the upper and lower bounds for the perimeters of the following rectangles.

(a)  

(b)

**Ans:** (a) The upper bounds of the rectangle are 4.25 m and 1.85 m. The upper bound of the perimeter = $2(4.25+1.85)=12.2$ m. The lower bounds of the rectangle are 4.15 m and 1.75 m. The lower bound of the perimeter = $2(4.15+1.75)=11.8$ m.
(b) The upper bounds of the rectangle are 110.5 m and 65.5 m.
The upper bound of the perimeter = 2(110.5+65.5)=352 m.
The lower bounds of the rectangle are 109.5 m and 64.5 m.
The lower bound of the perimeter = 2(109.5+64.5)=348 m.

Example: The plot of land represented by the following rectangle is to be fenced all round apart from a distance of about 50m, where there is already a wall. Calculate the maximum and minimum length of the fence needed.

Ans: Length of fencing = perimeter - length of wall. The minimum length of fencing is required when the perimeter is at its minimum value and the wall is at its maximum value.
So minimum fencing = 348 - 50.5 = 297.5 m.
Similarly, maximum fencing = upper bound perimeter - minimum wall.
Maximum fencing = 352 - 49.5 = 302.5 m.

(c) If a space is to be left in the fencing for gates measuring 2.3m, calculate the new upper and lower bounds for the amount of fencing needed.

Ans: New upper bound = max fencing - min size of gate
New upper bound = 302.5 - 2.25 m = 300.25 m
New lower bound = min fencing - upper bound size of gate
New lower bound = 297.5 - 2.35 m = 295.15 m

Example: If \( a = b - 2c \) find the upper and lower bounds for \( a \) when \( b = 4.8 \) (to 1 d.p) and \( c = 1.7 \) (to 1 d.p).

Ans: Upper bound \( a = 4.85 - 2 \times 1.65 = 1.55 \)
Lower bound \( a = 4.75 - 2 \times 1.75 = 1.25 \)

Note: One day of the school photo about 30 of the 927 pupils were absent. Find the upper and lower bounds for the number of pupils in the photo.

Ans: First find the upper and lower bounds for the number of pupils who were away: \( 25 \leq 30 \leq 34 \)

Now subtract these from the total:
Note:

Subtracting the minimum number of absent pupils gives the maximum number in the photo and vice versa.

Example:
(a) Of the 297 pupils about 500 are boys. Calculate the maximum number of girls.
(b) To the nearest 50, 750 pupils are under 16. Calculate the minimum number of pupils who are 16 or over.

Ans:
(a) 477  (b) 153

Lower bound = 4.75 - 2x1.75 = 1.25

Note:
A square has sides of 3 m. Find the possible values for its area.

$2.5 \leq \text{side length} \leq 3.5$

$(2.5)^2 \leq \text{area} \leq (3.5)^2$

$6.25 \ m^2 \leq \text{area} \leq 12.25 \ m^2$

So there is 1m between the upper and lower bounds for the sides of the square but the difference between the upper and lower bounds for the area is 6 $m^2$.

Example:
A lawn is 15 m long and 8 m wide to the nearest metre.

(a) Calculate the upper bound area of the lawn
(b) Calculate the lower bound area.
(c) The lawn is to be seeded at a rate of 30 g per square metre. Calculate the maximum amount of seed required.

Ans:
(a) Upper bound area = 15.5 x 8.5 = 131.75 $m^2$
(b) Lower bound area = 14.5 x 7.5 = 108.75 $m^2$
(c) Upper bound = $\frac{30 \ g}{m^2} \times 131.75 \ m^2 = 4018.4 \ g$

Note:
The ceiling of the room shown is to be painted. The instructions on the paint state that 1 litre of paint will cover 13 $m^2$. Calculate the upper and lower bounds of the amount of paint needed.

First you need to know the upper and lower bounds for the area of the ceiling.

Upper bound = 5.25 x 6.65 = 34.9125 $m^2$
Lower bound = 5.15 x 6.55 = 33.7325 $m^2$

The upper and lower bounds for the area covered by 1 litre of paint are 13.5 $m^2$ and 12.5 $m^2$. To calculate the amount of paint needed you must divide the area of the ceiling by the amount that 1 litre will cover. To calculate the maximum amount
needed you assume the worst: maximum ceiling area and minimum paint coverage.

\[
\frac{\text{Max area of ceiling}}{\text{min imum coverage}} = \frac{34.9125}{12.5} = 2.793 \text{ litres}
\]

Example: (a) Which measurements would you need to use to find the minimum amount of paint needed?

(b) Assuming the room has no doors or windows, calculate the minimum number of 1 litre tins needed to paint the walls.

(c) Again ignoring any doors or windows, what is the maximum number of 1 litre tins needed to paint the walls?

Ans: (a) To calculate the minimum amount of paint needed you divide the minimum ceiling area by the maximum paint coverage.

(b) Lower bound of wall area = \(2(2.05 \times 5.15) + 2(2.05 \times 6.55) = 47.97 \text{ m}^2\)

\[
\frac{47.97}{13.5} = 3.553 \text{ which rounds to 4 litres tins.}
\]

(c) Upper bound of wall area = \(2(2.15 \times 5.25) + 2(2.15 \times 6.65) = 51.17 \text{ m}^2\)

\[
\frac{51.17}{12.5} = 4.0936 \text{ which gives a maximum of 5 litres tins.}
\]

Note: If a weight is given as 10 grams to the nearest gram, then the actual weight will lie in the interval 9.5 grams to 10.499999... grams as all values in this interval will be rounded to 10 grams to the nearest gram. The weight 10.499999... grams is usually written as 10.5 g although it is accepted that 10.5 g would be rounded to 11 g (to the nearest gram).

The value 9.5 g is called the lower bound as it is the lowest value which would be rounded to 10 g while 10.5 g is called the upper bound.
Example: A rectangle measures 10 cm by 6 cm, where each measurement is given to the nearest cm. Write down an interval approximation for the area of a rectangle.

(a) The lower bound (minimum area) = $9.5 \times 5.5 = 52.25 \text{ cm}^2$
(b) The upper bound (maximum area) = $10.5 \times 6.5 = 68.25 \text{ cm}^2$

The interval approximation = $52.25 \text{ cm}^2$ to $68.25 \text{ cm}^2$

Example: To the nearest whole number, the value of $p = 215$ and the value of $q = 5$. Calculate the maximum and minimum values of the following expressions.

(a) $p + q$  (b) $p - q$  (c) $p \times q$  (d) $\frac{p}{q}$

Ans:

$p_{\text{min}} = 214.5$  \hspace{1cm} $p_{\text{max}} = 215.5$
$q_{\text{min}} = 4.5$  \hspace{1cm} $q_{\text{max}} = 5.5$

(a) For $p + q$  \hspace{1cm} maximum = $215.5 + 5.5 = 221$
minimum = $214.5 + 4.5 = 219$

(b) For $p - q$  \hspace{1cm} maximum = $215.5 - 4.5 = 211$
minimum = $214.5 - 5.5 = 209$

(c) For $p \times q$  \hspace{1cm} maximum = $215.5 \times 5.5 = 1185.25$
minimum = $214.5 \times 4.5 = 965.25$

(d) For $\frac{p}{q}$  \hspace{1cm} maximum = $\frac{215.5}{4.5} = 47.888888 ...$
minimum = $\frac{215.5}{5.5} = 39$

Example: Ahmad time of 239.2 seconds is known to be correct to the nearest tenth of a second. What is the shortest time that it could actually be?

Ans: If 239.2 seconds is correct to the nearest tenth of a second then the shortest time it could be is 239.15 seconds.

Example: One car costs £18700 and another costs £17300, both prices being given to the nearest £100. What is the least possible difference in price between the two cars?

Ans: As each price is given to the nearest £100 then:
1st car at £18700: maximum = £18749.99
Note that £18750 would be rounded up to £18800
Minimum = £18650.00

2nd car at £17300: maximum = £17349.99
Note that £17350 would be rounded up to £17400
Minimum = £17250.00

Least possible difference in price = 1st Min - 2nd Max = £18650 - £17349.99 = £1300.01

Note: You must always take care when working with money or ages to identify lower and upper bounds.

(b) The upper bound (maximum area) = 10.5 \times 6.5 = 68.25 \text{ cm}^2
The interval approximation = 52.25 \text{ cm}^2 to 68.25 \text{ cm}^2

Note: When you measure a length, you will always find an approximate answer. If your garden path is 5.218174 m long, you will say its length is 5 m or 5.2 m or even possibly 5 m 22 cm. Hence, if your friend says his garden path is 7 m long, you will assume that the length is 7 m to the nearest metre.
The actual length of the path can be anything from 6.5 m to 7.4999... m (which is virtually 7.5 m).
The lower bound is the smallest value which the number could be (in this case 6.5 m), and the upper bound is the largest value which the number could be (in this case 7.5 m).

Example: A weight is 17 kg correct to the nearest integer. What are its upper and lower bounds?

Ans: The upper bound is 17.5 kg; the lower bound is 16.5 kg.

Example: A tree is 12.3 m high (to the nearest 0.1 of a metre). What are the upper and lower bounds for its height?

Ans: The upper bound is 12.35 m; the lower bound is 12.25 m.

In some cases, e.g. in money, the value given goes up in steps.
For example: an item priced at £3.60 to the nearest 10p can have an actual value between £3.55 and £3.64.
This is because:
(i) an item cannot be priced at £3.641 (64.1 pence does not exist)
(ii) a price of £3.65 would round up to £3.70.
Hence £3.64 is its largest possible price.
Thus if the measurement can take all possible values, a measurement of 12 to the nearest integer can have an actual value between 11.5 and 12.5.
If the measurement goes up in steps, a measurement of 12.5 would be rounded to 13. Whenever 12.4999 is a possible measurement, the data is continuous.
Although age is continuous, people give their age in steps of one day. Age is recorded as going from, for example, 17 years 0 days to 17 years 1 day. Thus age can be treated as a discrete variable.

**Example:** Victoria is aged 17. What are the upper and lower bounds of her age?

**Ans:** The exact age of a person aged 17 years can be between 17 years 0 days and 17 years 364 days (unless it is a leap year!) Hence, the upper bound is 17 years 364 days and the lower bound is 17 years.

**Example:** Give the lower and upper bounds for:
1. The length of a lorry measured as 12 m to the nearest metre
2. The length of a car measured as 4.7 m to the nearest 0.1 m
3. The weight of a lorry measured as 38 tons to the nearest ton
4. The cost of a TV given as £290 to the nearest £10
5. The age of Caroline who is 16 years old (it is not a leap year)
6. The weight of a packet of cornflakes given as 350 grams to the nearest 5 grams.

**Calculations Involving Lower or Upper Bounds**

In cases where two or more values are combined, you must consider carefully whether or not the values have been rounded. You may need to use the upper bounds of these values to calculate the upper bound of the combined values, and vice versa, or possibly a combination of both the upper and lower bounds (see example 4).

**Example:** A rectangular field is 25 m by 15 m (each measurement to the nearest metre).
What are the lower and upper bounds for its area?
Lower bound for its area = Minimum possible length x Minimum possible width
The minimum possible length of the field is 24.5 m
The minimum possible width of the field is 14.5 m
The minimum possible area is $24.5 \times 14.5 = 355.25 \text{ m}^2$
Therefore, the lower bound of the area is $355.25 \text{ m}^2$

Upper bound for its area = Maximum possible length x Maximum possible width
The upper bounds for the dimensions are 25.5 and 15.5 m.

Therefore, the upper bound for the area $25.5 \times 15.5 = 395.25 \text{ m}^2$
**LESSON 13**

**Simplify**

Remember **BODMAS** = Bracket, Of, Division, Multiplication, Add, Subtract

Example: Evaluate: $2 \times 3 + 4 = 6 + 4 = 10$

Example: Evaluate: $6 \div 2 - 4 = 3 - 4 = -1$

Example: Evaluate: $2 \times (3 + 4) = 2 \times 7 = 14$

Example: Evaluate: $12 \times 2 \div 4 \times 3 = 12 \times \left(\frac{2}{4}\right) \times 3 = 18$

Example: Evaluate: $27 \div (7 - 4) + 3 \times 4 = 27 \div 3 + 3 \times 4 = \left(\frac{27}{3}\right) + 3 \times 4 = 9 + 3 \times 4 = 9 + 12 = 21$

Example: Evaluate: $[(6 \times 2) \div (1 + 2)] \div [3 + 5 - 7 + 3] = [12 \div 3] \div [4] = 4 \div 4 = 1$

Example: (a) Simplify $a^4 \times a^5 = a^{4+5} = a^9$ (1 Mark)

(b) Simplify $4xy^3 \times 3x^2 y = 12x^3 y^4$ (2 Marks)

(c) Factorise $p^2 - 16q^2 = (p + 4q)(p - 4q)$ (2 Marks)

Simplify $(2x^3 y)^5 = 2^5 \times x^{15} \times y^5 = 32x^{15} y^5$ (2 Marks)

(b) Simplify $\frac{x^2 - 4x}{x^2 - 6x + 8} = \frac{x(x - 4)}{(x - 4)(x - 2)} = \frac{x}{x - 2}$ (3 Marks)
Additional Questions

1. \( a^3 \times a^5 = a^8 \) 
2. \( a^3 \times a^{-5} = a^{-2} = \frac{1}{a^2} \)

3. \( a^p \times a^b = a^{p+b} \) 
4. \( a^c \times a^{-d} = a^{c-d} \)

5. \( \frac{a^3 \times a^5}{b^2 \times b^3} = \frac{a^8}{b^5} \) 
6. \( \frac{a^3}{a^2} = a \)

7. \( \frac{a^3}{a^2} = a^{3-(-2)} = a^5 \) 
8. \( \frac{a^{-3}}{a^{-2}} = a^{-3-(-2)} = a^{-1} = \frac{1}{a} \)

**LESSON 14**

**The reciprocal of a number**

The reciprocal of a number is found by inverting it. For example, the reciprocal of \( \frac{2}{5} \) is \( \frac{5}{2} \).

1. Find the reciprocal of each of the following expressions:
   (a) \( \frac{x}{y} \) 
   (b) \( \frac{x+2}{y+5} \) 
   (c) \( \frac{1}{y+5} \)
   (d) \( 5y \) 
   (e) \( x-5 \) 
   (f) \( \frac{1}{3y} \)
   (g) \( \sqrt{y} \) 
   (h) \( \sqrt{x-5} \) 
   (i) \( \frac{y-5}{3y} \)

Ans:
   (a) \( \frac{y}{x} \) 
   (b) \( \frac{y+5}{x+2} \) 
   (c) \( \frac{y+5}{1} = y+5 \)
   (d) \( \frac{1}{5y} \) 
   (e) \( \frac{1}{x-5} \) 
   (f) \( \frac{3y}{1} = 3y \)
   (g) \( \frac{1}{\sqrt{y}} \) 
   (h) \( \frac{1}{\sqrt{x-5}} \) 
   (i) \( \frac{3y}{y-5} \)

2. Make \( t \) the subject of the formula
\[ D = 5t + \pi t + 5w \]

**Ans:**

\[ 5t + \pi t = D - 5w \]
\[ (5 + \pi)t = D - 5w \]
\[ t = \frac{D - 5w}{5 + \pi} \]

3. **The following formula is used in industry:**

\[ P = mt - g \]

(a) **Calculate** \( P \) **when** \( m = 45 \), \( t = 0.3 \) and \( g = -5 \)

**Ans:**

\[ P = mt - g \]
\[ P = 45 \times 0.3 - (-5) = 13.5 + 5 = 18.5 \]

(b) **Rearrange the formula to express** \( t \) **in terms of** \( P \), \( m \) and \( g \)

**Ans:**

\[ P = mt - g \]
\[ mt = P + g \]
\[ t = \frac{P + g}{m} \]

(c) **Calculate** \( t \) **when** \( P = 15.9 \), \( m = 0.75 \) and \( g = -4 \)

**Ans:**

\[ t = \frac{P + g}{m} = \frac{15.9 - 4}{0.75} = \frac{11.9}{0.75} = 15.9 \quad (1 \text{ dp}) \]

4. **Make** \( a \) **the subject of** \( r = 3a + 6 \)

**Ans:**

\[ r = 3a + 6 \]
\[ 3a = r - 6 \]
\[ a = \frac{r - 6}{3} \]
\[ a = \frac{r}{3} - 2 \]

5. **Make** \( b \) **the subject of** \( a = 3b^2 - 8 \)

**Ans:**

\[ a = 3b^2 - 8 \]
\[ 3b^2 = a + 8 \]
\[ b^2 = \frac{a+8}{3} \]
\[ b = \pm \sqrt{\frac{a+8}{3}} \]

6. Make \( a \) the subject of \( b = \pm \sqrt{\frac{a+8}{3}} \)

Ans: \( b = \pm \sqrt{\frac{a+8}{3}} \)

\[ b^2 = \left( \pm \sqrt{\frac{a+8}{3}} \right)^2 \]
\[ b^2 = \frac{a+8}{3} \]
\[ 3b^2 = a+8 \]
\[ a = 3b^2 - 8 \]

7. Make \( y \) the subject of \( x = \frac{21-5y}{2} \)

Ans: \( x = \frac{21-5y}{2} \)
\[ 2x = 21-5y \]
\[ 5y = 21-2x \]
\[ y = \frac{21-2x}{5} \]

8. Make \( z \) the subject of \( x = a \left( z - \frac{b}{c} \right) \)

Ans: \( x = a \left( z - \frac{b}{c} \right) \)
\[ x = az - \frac{ab}{c} \]
\[ az = x + \frac{ab}{c} \]
\[ z = \frac{x + \frac{ab}{c}}{a} \]
\[ z = \frac{x}{a} + \frac{b}{ac} \]

9. Make \( r \) the subject of \( V = \frac{4}{3} \pi r^3 \)

Ans: \( V = \frac{4}{3} \pi r^3 \)
10. The volume of a cone is given by the formula \( V = \frac{1}{3} \pi r^2 h \). Make \( r \) the subject of this formula.

\[
V = \frac{1}{3} \pi r^2 h \\
\Rightarrow r^2 = \frac{3V}{\pi h} \\
\Rightarrow r = \pm \sqrt[3]{\frac{3V}{\pi h}}
\]

11. Make \( r \) the subject of \( \sqrt{V} = \frac{4}{3} \pi r^3 \)

\[
\sqrt{V} = \frac{4}{3} \pi r^3 \\
(\sqrt{V})^3 = \left( \frac{4}{3} \pi r^3 \right)^2 \\
V = \frac{16}{9} \pi^2 r^6 \\
r^6 = \frac{9V}{16\pi^2} \\
r = \sqrt[6]{\frac{9V}{16\pi^2}}
\]

12. Make \( z \) the subject of \( s = \frac{z+1}{z-1} \)

\[
s = \frac{z+1}{z-1} \\
sz - s = z + 1 \\
sz - z = s + 1 \\
z(s - 1) = s + 1 \\
z = \frac{s + 1}{s - 1}
\]

13. Make \( x \) the subject in the formula \( v = \omega \sqrt{a^2 - x^2} \)

\[
v = \omega \sqrt{a^2 - x^2}
\]
\[ v^2 = \left(\omega \sqrt{a^2 - x^2}\right)^2 \]
\[ v^2 = \omega^2 \left(a^2 - x^2\right) \]
\[ v^2 = \omega^2 a^2 - \omega^2 x^2 \]
\[ \omega^2 x^2 = \omega^2 a^2 - v^2 \]
\[ x^2 = \frac{\omega^2 a^2 - v^2}{\omega^2} = a^2 - \frac{v^2}{\omega^2} \]
\[ x = \pm \sqrt{a^2 - \frac{v^2}{\omega^2}} \]

14. Make \( u \) the subject in \( f = \frac{uv}{u+v} \)

Ans: \[ f = \frac{uv}{u+v} \]
\[ f(u+v) = uv \]
\[ f u + f v = uv \]
\[ f u - uv = - f v \]
\[ u(f - v) = - f v \]
\[ u = \frac{- f v}{f - v} = \frac{f v}{v - f} \]

15. Make \( R \) the subject in \( \frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} \)

Ans: \[ \frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} \]
\[ 1 = \frac{R_2 + R_1}{R_1 R_2} \]
\[ \frac{1}{R} = \frac{R_2 + R_1}{R_1 R_2} \]
\[ R = \frac{R_2 R_1}{R_2 + R_1} \]

The subject of a formula is the variable that stands alone on one side of the formula.

Examples:
\[ s = \frac{d}{t} \quad s \text{ is the subject} \]
\[ t = \frac{d}{s} \quad t \text{ is the subject} \]
\[ d = st \quad d \text{ is the subject} \]

Example: Make \( y \) the subject of the formula:
\[ a = b(x + y) \]

**Ans:**
\[ a = bx + by \]
\[ by = a - bx \]
\[ y = \frac{a - bx}{b} \]

**Example:** Make \( y \) the subject of the formula:

\[ y(\pi + 2) = r \]

**Ans:**
\[ y = \frac{r}{\pi + 2} \]

**Example:** The formula for the surface area, \( A \), of a closed circular cylinder of radius \( r \) and height \( h \) is:

\[ A = 2\pi(r + h) \]

(a) Make \( h \) the subject of the formula.

**Ans:**
\[ A = 2\pi(r + h) \]
\[ A = 2\pi r^2 + 2\pi rh \]
\[ 2\pi rh = A - 2\pi r^2 \]
\[ h = \frac{A - 2\pi r^2}{2\pi} \]

(b) Write down the equation if the surface area is \( 120\pi \text{ cm}^2 \) and the height is \( 12 \text{ cm} \).

**Ans:**
\[ A = 2\pi(r + h) \]
\[ 120\pi = 2\pi(r + 12) \]
\[ 120\pi = 2\pi r^2 + 24\pi \]

Example: Make \( x \) the subject of the formula \( y = 3x + 2 \).

**Ans:**
\[ y = 3x + 2 \]
\[ 3x = 2 - y \]
\[ x = \frac{2 - y}{3} \]

Example: Make \( b \) the subject of the formula \( a = \frac{m}{b^2} \).

**Ans:**
\[ a = \frac{m}{b^2} \]
\[ b^2 = \frac{m}{a} \]
\[ \Rightarrow b = \sqrt{\frac{m}{a}} \]
LESSON 15

Prime Numbers

A prime number is a whole number that has only two factors: itself and 1. A prime number is only divisible by itself and 1. For example, 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 51, 53, 59, 61, 67, 71, 73, 79, 83, 89, 91, 97 are the examples of prime numbers in the list of numbers from 0 to 100.

Prime Factors

Special Number Sequences

There are FIVE special sequences of numbers that you should know at this stage.

1. **EVEN Numbers:** All those numbers whose first digit is 0 or even.
   Examples: 10, 12, 34, 20, -48, -42, ... and so on

2. **ODD Numbers:** All those numbers whose first digit is odd.
   Examples: 11, 35, 27, -49, -43, ... and so on

Note:

3. **SQUARE Numbers:**

   
   \[
   \begin{array}{cccccccc}
   (1\times1) & (2\times2) & (3\times3) & (4\times4) & (5\times5) & (6\times6) & (7\times7) & (8\times8) & (9\times9) & (10\times10) \\
   1 & 4 & 9 & 16 & 25 & 36 & 49 & 64 & 81 & 100 \\
   \end{array}
   \]
4. **CUBE Numbers:**

\[
\begin{array}{cccccccccc}
(1x1x1) & (2x2x2) & (3x3x3) & (4x4x4) & (5x5x5) & (6x6x6) & (7x7x7) & (8x8x8) & (9x9x9) & (10x10x10) \\
1 & 8 & 27 & 64 & 125 & 216 & 343 & 512 & 729 & 1000
\end{array}
\]

5. **TRIANGLE Numbers:**

\[
\begin{array}{cccccccccc}
1 & 3 & 6 & 10 & 15 & +2 & +3 & +4 & +5 \\
\end{array}
\]

**Equivalent Fractions**

\[
\begin{align*}
\frac{1}{2} &= \frac{2}{4} = \frac{3}{6} = \frac{4}{8} = \frac{5}{10} = \frac{6}{12} = \text{Any number except zero} \\
\text{Twice that number} \\
\frac{1}{5} &= \frac{2}{10} = \frac{3}{15} = \frac{4}{20} = \frac{5}{25} = \frac{6}{30} = \text{Any number except zero} \\
\text{Five times that number} \\
\frac{1}{9} &= \frac{2}{18} = \frac{3}{27} = \frac{4}{36} = \frac{5}{45} = \frac{6}{54} = \text{Any number except zero} \\
\text{Nine times that number}
\end{align*}
\]

**Cancelling Down**

\[
\begin{align*}
\frac{3}{15} \div 3 &= \frac{1}{5} \\
\frac{6}{15} \div 3 &= \frac{2}{5}
\end{align*}
\]
Example: Find the highest common factor (HCF) and the lowest common multiple (LCM) of the numbers 8 and 12.

Ans:

Factors of 8 are: 1, 8, 2, 4

Factors of 12 are: 1, 12, 2, 6, 3, 4

Common Factors are: 1, 2, 4

Therefore, HCF = 4

Multiples of 8 are: 8, 16, 24, 32, 40, 48, 56, ...

Multiples of 12 are: 12, 24, 36, 48, 60, 72, ...
Common multiples are: 24, 48, 72, ...

Therefore, LCM = 24

Write the number 3960 as a product of its prime factors

\[
\begin{align*}
3960 \div 2 &= 1980 \\
1980 \div 2 &= 990 \\
990 \div 2 &= 495 \\
495 \div 3 &= 165 \\
165 \div 3 &= 55 \\
55 \div 5 &= 11 \\
11 \div 11 &= 1
\end{align*}
\]

\[3960 = 2 \times 2 \times 2 \times 3 \times 3 \times 5 \times 11\]

10  (a) Write 72 as a product of prime factors.
(b) Find the lowest common multiple (LCM) of 72 and 108
(c) Find the highest common factor (HCF) of 72 and 108

Ans:

\[
\begin{align*}
72 & \div 2 = 36 \\
36 & \div 2 = 18 \\
18 & \div 2 = 9 \\
9 & \div 3 = 3 \\
3 & \div 3 = 1
\end{align*}
\]

\[
\begin{align*}
72 &= 2 \times 2 \times 2 \times 3 \times 3 \\
108 &= 2 \times 2 \times 3 \times 3 \times 3
\end{align*}
\]

\[\text{LCM} = 2 \times 2 \times 2 \times 3 \times 3 \times 3 = 216\]

\[\text{HCF} = 2 \times 2 \times 3 \times 3 = 36\]

**Multiples of Numbers**

Example: The multiples of 2 are:

\[1 \times 2 = 2 \quad 2 \times 2 = 4 \quad 3 \times 2 = 6 \quad 4 \times 2 = 8\]

\[5 \times 2 = 10 \quad 6 \times 2 = 12 \quad \ldots \text{and so on}\]

Example: The multiples of 3 are:

\[1 \times 3 = 3 \]
\[2 \times 3 = 6\]
### Multiples of 3

Example: Some of the multiples of 3 are:

<table>
<thead>
<tr>
<th>1st multiple</th>
<th>2nd multiple</th>
<th>3rd multiple</th>
<th>4th multiple</th>
<th>5th multiple</th>
<th>100th multiple</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 x 3 = 3</td>
<td>2 x 3 = 6</td>
<td>3 x 3 = 9</td>
<td>4 x 3 = 12</td>
<td>5 x 3 = 15</td>
<td>... 100 x 3 = 300</td>
</tr>
</tbody>
</table>

... and so on

### Multiples of 4

Example: Some of the multiples of 4 are:

<table>
<thead>
<tr>
<th>1st multiple</th>
<th>2nd multiple</th>
<th>3rd multiple</th>
<th>4th multiple</th>
<th>5th multiple</th>
<th>100th multiple</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 x 4 = 4</td>
<td>2 x 4 = 8</td>
<td>3 x 4 = 12</td>
<td>4 x 4 = 16</td>
<td>5 x 4 = 20</td>
<td>... 100 x 4 = 400</td>
</tr>
</tbody>
</table>

... and so on

### Multiples of a

Example: Some of the multiples of a are:

<table>
<thead>
<tr>
<th>1st multiple</th>
<th>2nd multiple</th>
<th>3rd multiple</th>
<th>4th multiple</th>
<th>5th multiple</th>
<th>100th multiple</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 x a = a</td>
<td>2 x a = 2a</td>
<td>3 x a = 3a</td>
<td>4 x a = 4a</td>
<td>5 x a = 5a</td>
<td>... 100 x a = 100a</td>
</tr>
</tbody>
</table>

... and so on

### Multiples of b

Example: Some of the multiples of b are:

<table>
<thead>
<tr>
<th>1st multiple</th>
<th>2nd multiple</th>
<th>3rd multiple</th>
<th>4th multiple</th>
<th>5th multiple</th>
<th>100th multiple</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 x b = b</td>
<td>2 x b = 2b</td>
<td>3 x b = 3b</td>
<td>4 x b = 4b</td>
<td>5 x b = 5b</td>
<td>... 100 x b = 100b</td>
</tr>
</tbody>
</table>

... and so on

### Multiples of 0.1

Example: Some of the multiples of 0.1 are:

<table>
<thead>
<tr>
<th>1st multiple</th>
<th>2nd multiple</th>
<th>3rd multiple</th>
<th>4th multiple</th>
<th>5th multiple</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 x 0.1 = 0.1</td>
<td>2 x 0.1 = 0.2</td>
<td>3 x 0.1 = 0.3</td>
<td>4 x 0.1 = 0.4</td>
<td>5 x 0.1 = 0.5</td>
</tr>
</tbody>
</table>
Example: Some of the multiples of \(\frac{1}{10}\) are:

\[
\begin{align*}
1 \times \frac{1}{10} &= \frac{1}{10} \\
2 \times \frac{1}{10} &= \frac{2}{10} \\
3 \times \frac{1}{10} &= \frac{3}{10} \\
4 \times \frac{1}{10} &= \frac{4}{10} \\
5 \times \frac{1}{10} &= \frac{5}{10}
\end{align*}
\]

1st multiple 2nd multiple 3rd multiple 4th multiple 5th multiple

Example: Some of the multiples of \(\frac{1}{a}\) are:

\[
\begin{align*}
1 \times \frac{1}{a} &= \frac{1}{a} \\
2 \times \frac{1}{a} &= \frac{2}{a} \\
3 \times \frac{1}{a} &= \frac{3}{a} \\
4 \times \frac{1}{a} &= \frac{4}{a} \\
5 \times \frac{1}{a} &= \frac{5}{a}
\end{align*}
\]

1st multiple 2nd multiple 3rd multiple 4th multiple 5th multiple

\textbf{LESSON 16}

\textbf{FRACTION}

\textbf{FRACTION:} \quad \frac{\text{Numerator}}{\text{Denominator}} = \frac{2}{5} \text{ is an example of a fraction.}

\textbf{FRACTION:} \quad \frac{\text{Numerator}}{\text{Denominator}} = \frac{a}{b} \text{ is an example of a fraction.}

\textbf{FRACTION:} \quad \frac{\text{Numerator}}{\text{Denominator}} = \frac{x}{y} \text{ is an example of a fraction.}

\textbf{FRACTION:} \quad \frac{\text{Numerator}}{\text{Denominator}} = \frac{1}{x+5} \text{ is an example of a fraction.}

\textbf{FRACTION:} \quad \frac{\text{Numerator}}{\text{Denominator}} = \frac{a+b}{c+d} \text{ is an example of a fraction.}
TYPES OF FRACTIONS:

Example: \( \frac{2}{5} \) is an example of a **proper fraction**.

Example: \( \frac{9}{11} \) is an example of a **proper fraction**.

Example: \( \frac{a-1}{a} \) is an example of a **proper fraction**, provided \( a \neq 0 \).

Example: \( \frac{6}{5} \) is an example of an **improper fraction**.

Example: \( \frac{19}{11} \) is an example of an **improper fraction**.

Example: \( \frac{a+1}{a} \) is an example of an **improper fraction**, provided \( a \neq 0 \).

Example: Convert \( 3 \frac{2}{5} \) to an **improper fraction**.

**Ans:** \( 3 \frac{2}{5} = \frac{17}{5} \)

Example: Convert \( 4 \frac{1}{3} \) to an **improper fraction**.

**Ans:** \( 4 \frac{1}{3} = \frac{13}{3} \)

Example: Convert \( 3 \frac{3}{4} \) to an **improper fraction**.

**Ans:** \( 3 \frac{3}{4} = \frac{3 \times 4 + 3}{4} = \frac{12 + 3}{4} = \frac{15}{4} = 3 + \frac{3}{4} \)

Example: Convert \( \frac{a - c}{b} \) to an **improper fraction**.

**Ans:** \( \frac{a - c}{b} = \frac{a \times b + c}{b} = \frac{ab + c}{b} = a + \frac{c}{b} \)

Example: Convert \( \frac{19}{6} \) to a **mixed number**.

**Ans:** \( \frac{19}{6} = 3 \frac{1}{6} \)
Example: Convert \( \frac{141}{22} \) to a mixed number.

Ans: \( \frac{141}{22} = 6 \frac{9}{22} \)

Example: Convert the improper fraction \( \frac{132}{15} \) to a mixed number, in their lowest terms:

Ans: \( \frac{132}{15} = 8 \frac{12}{15} = 8 \frac{4}{5} \)

Example: Convert \( 13 \frac{2}{5} \) to an improper fraction. Ans: \( 13 \frac{2}{5} = \frac{67}{5} \)

Adding Fractions

Example: Add the following fractions.

(a) \( \frac{3}{4} + \frac{2}{4} = \frac{3 + 2}{4} = \frac{5}{4} \)  
(b) \( \frac{5}{7} + \frac{3}{7} = \frac{5 + 3}{7} = \frac{8}{7} \)  
(c) \( \frac{11}{14} + \frac{2}{14} = \frac{11 + 2}{14} = \frac{13}{14} \)

Example: Add the following fractions.

(a) \( 1 \frac{3}{4} + 2 \frac{2}{4} = \frac{7 + 10}{4} = \frac{17}{4} = 4 \frac{1}{4} \)
(b) \( 3 \frac{5}{7} + 2 \frac{3}{7} = \frac{26 + 17}{7} = \frac{43}{7} = 6 \frac{1}{7} \)
(c) \( 1 \frac{1}{4} + 2 \frac{2}{3} = \frac{5 + 8}{4} = \frac{13 + 32}{12} = \frac{47}{12} = 3 \frac{11}{12} \)

Example: Add the following fractions.

(a) \( \frac{a + c}{b} \cdot \frac{a - c}{d} = \frac{ad - bc}{bd} \)
(b) \( \frac{a + c}{b} \cdot \frac{e}{f} = \frac{adf + bcf - bde}{bdf} \)
(c) \( \frac{a - c}{b} \cdot \frac{e}{f} = \frac{adf - bcf + bde}{bdf} \)

Example: Evaluate the following fractions.

(a) \( \frac{5}{8} - \frac{3}{4} + \frac{4}{5} = \frac{25}{40} - \frac{30}{40} + \frac{32}{40} = \frac{25 - 30 + 32}{40} = \frac{27}{40} \)

(b) \( 3 \frac{5}{7} + 2 \frac{3}{5} - \frac{1}{3} = \frac{26 + 13 - 4}{105} = \frac{15 \times 26 + 21 \times 13 - 5 \times 4}{105} = \frac{390 + 273 - 140}{105} = \frac{523}{105} = 4 \frac{103}{105} \)

(c) \( 1 \frac{1}{4} - \frac{2}{3} = \frac{5}{12} - \frac{8}{12} = \frac{15 - 8}{12} = \frac{7}{12} \)
Subtracting Fractions

Example: Subtract the following fractions.

(b) \( \frac{3}{4} \) \(-\) \( \frac{2}{4} = \frac{3-2}{4} = \frac{1}{4} \)

(b) \( \frac{5}{7} \) \(-\) \( \frac{3}{7} = \frac{5-3}{7} = \frac{2}{7} \)

(c) \( \frac{11}{14} \) \(-\) \( \frac{2}{14} = \frac{11-2}{14} = \frac{9}{14} \)

Multiplying Fractions

Example: Multiply the following fractions.

(a) \( \frac{20}{5} \times \frac{6}{4} = \frac{20\times6}{5\times4} = \frac{120}{20} = \frac{6}{1} \)

(b) \( \frac{56}{7} \times \frac{15}{8} = \frac{56\times15}{7\times8} = \frac{840}{56} = \frac{15}{1} \)

(c) \( \frac{42}{22} \times \frac{11}{14} = \frac{42\times11}{22\times14} = \frac{462}{308} = \frac{3}{2} \)

Dividing Fractions

Example: Divide the following fractions.

(c) \( \frac{3}{4} \div \frac{2}{5} = \frac{3\times5}{4\times2} = \frac{15}{8} \)

(b) \( \frac{5}{7} \div \frac{3}{8} = \frac{5\times8}{7\times3} = \frac{40}{21} \)

(c) \( \frac{11}{14} \div \frac{2}{3} = \frac{11\times3}{14\times2} = \frac{33}{28} \)

Cancelling Fractions Down

Example: Reduce the following fractions.

\( \frac{8}{16} \div 8 = \frac{8\div8}{16\div8} = \frac{1}{2} \)

(b) \( \frac{3}{21} \div 3 = \frac{3\div3}{21\div3} = \frac{1}{7} \)

(c) \( \frac{5}{15} \div 5 = \frac{5\div5}{15\div5} = \frac{1}{3} \)

Example

\( \frac{1+3}{4} = \frac{2\times1+1\times3}{4} = \frac{2+3}{4} = \frac{5}{4} = \frac{1}{4} \)

Example

\( \frac{3+5}{7} = \frac{7\times3+4\times5}{28} = \frac{21+20}{28} = \frac{41}{28} = 1\frac{13}{28} \)

Example

\( \frac{5+3}{8} = \frac{7\times5+8\times3}{56} = \frac{35+24}{56} = \frac{59}{56} = 1\frac{3}{56} \)

Example

\( a + c = \frac{d\times a + b \times c}{bd} = \frac{da + bc}{bd} = \frac{ad + bc}{bd} \)

Example

\( a + y = \frac{z\times a + b \times y}{bz} = \frac{za + by}{bz} = \frac{az + by}{bz} \)

Example

\( \frac{\frac{1}{2} + \frac{3}{4} + \frac{3}{2} + 11}{4} = \frac{2\times3+1\times11}{4} = \frac{6+11}{4} = \frac{17}{4} = 4\frac{1}{4} \)
Example: \[
\frac{3}{4} + \frac{5}{7} = \frac{15}{4} + \frac{19}{7} = \frac{7 \times 15 + 4 \times 19}{28} = \frac{105 + 76}{28} = \frac{181}{28} = 6\frac{13}{28}
\]

Example: \[
\frac{1}{2} - \frac{3}{4} = \frac{2 \times 1 - 1 \times 3}{4} = \frac{2 - 3}{4} = -\frac{1}{4}
\]

Example: \[
\frac{3}{4} - \frac{5}{7} = \frac{7 \times 3 - 4 \times 5}{28} = \frac{21 - 20}{28} = \frac{1}{28}
\]

Example: \[
\frac{5}{8} - \frac{3}{7} = \frac{7 \times 5 - 8 \times 3}{56} = \frac{35 - 24}{56} = \frac{11}{56}
\]

Example: \[
1 - \frac{2}{3} - \frac{3}{4} - \frac{11}{4} = \frac{2 \times 3 - 1 \times 11}{4} = \frac{6 - 11}{4} = -\frac{5}{4} = -1\frac{1}{4}
\]

Example: \[
\frac{3}{4} - \frac{5}{7} = \frac{7 \times 15 - 4 \times 19}{28} = \frac{105 - 76}{28} = \frac{29}{28} = 1\frac{1}{28}
\]

Example: \[
a \cdot c = \frac{d \times a - b \times c}{bd} = \frac{da - bc}{bd} = \frac{ad - bc}{bd}
\]

Example: \[
a \cdot y = \frac{z \times a - b \times y}{bz} = \frac{za - by}{bz} = \frac{az - by}{bz}
\]

Example: \[
\frac{1}{2} \times \frac{3}{4} = \frac{1 \times 3}{2 \times 4} = \frac{3}{8}
\]

Example: \[
\frac{3}{4} \times \frac{5}{7} = \frac{3 \times 5}{4 \times 7} = \frac{15}{28}
\]

Example: \[
\frac{5}{8} \times \frac{3}{7} = \frac{5 \times 3}{8 \times 7} = \frac{15}{56}
\]

Example: \[
\frac{3}{4} \times \frac{5}{7} = \frac{11}{4} \times \frac{12}{7} = \frac{11 \times 12}{4 \times 7} = \frac{132}{28}
\]

Example: \[
\frac{5}{8} \times \frac{3}{7} = \frac{13}{8} \times \frac{17}{7} = \frac{13 \times 17}{8 \times 7} = \frac{221}{56}
\]

Example: \[
\frac{a}{b} \times \frac{c}{d} = \frac{a \times c}{b \times d} = \frac{ac}{bd}
\]

Example: \[
\frac{a}{b} \times \frac{y}{z} = \frac{a \times y}{b \times z} = \frac{ay}{bz}
\]
**Example**

\[
\frac{1}{2} + \frac{3}{4} = \frac{1 \times 4}{2 \times 3} = \frac{1 \times 4}{6} = \frac{4}{6} = \frac{1}{3}
\]

\[
\frac{5}{8} \div \frac{3}{11} = \frac{5 \times 11}{8 \times 3} = \frac{55}{24} = 2 \frac{7}{24}
\]

\[
\frac{7}{9} \div \frac{5}{2} = \frac{7 \times 5}{9 \times 2} = \frac{35}{18} = 1 \frac{17}{18}
\]

\[
1 \frac{5}{8} + \frac{3}{11} = \frac{13}{8} + \frac{25}{11} = \frac{13 \times 11}{8 \times 25} = 1 \frac{143}{200}
\]

\[
\frac{2}{9} + 1 \frac{2}{5} = \frac{25}{9} + \frac{7}{5} = \frac{25 \times 5}{9 \times 7} = \frac{125}{63} = 1 \frac{62}{63}
\]

\[
a \div c = \frac{a}{b} \div \frac{c}{d} = \frac{a \times d}{b \times c} = \frac{ad}{bc}
\]

\[
a \div y = \frac{a}{b} \div \frac{y}{z} = \frac{a \times z}{b \times y} = \frac{az}{by}
\]

\[
\frac{2}{x+5} + \frac{3}{x-3} = \frac{2(x-3)+3(x+5)}{(x+5)(x-3)} = \frac{2x-6+3x+15}{(x+5)(x-3)} = \frac{5x+9}{(x+5)(x-3)}
\]

\[
\frac{2}{x-5} - \frac{3}{x+3} = \frac{2(x+3)-3(x-5)}{(x-5)(x+3)} = \frac{2x+6-3x+15}{(x-5)(x+3)} = \frac{-x+21}{(x-5)(x+3)} = \frac{21-x}{(x-5)(x+3)}
\]

\[
\frac{x+2}{5} - \frac{x-3}{4} = \frac{4(x+2)-5(x-3)}{20} = \frac{4x+8-5x+15}{20} = \frac{-x+23}{20} = \frac{23-x}{20}
\]

\[
\frac{2}{x-5} \times \frac{3}{x+3} = \frac{2 \times 3}{(x-5)(x+3)} = \frac{6}{(x-5)(x+3)}
\]

\[
\frac{x+2}{x+5} + \frac{x-3}{x-7} = \frac{x+2}{x+5} \times \frac{x-7}{x-3} = \frac{(x+2)(x-7)}{(x+5)(x-3)}
\]

**Exercise**

1. Give an example of a **proper fraction**.
   
   Ans: \( \frac{3}{5} \) is an example of a **proper fraction**

2. Give an example of an **improper fraction**.
   
   Ans: \( \frac{2}{5} \) is an example of an **improper fraction**
3. Give an example of a **mixed fraction**.

   **Ans:** $3\frac{2}{5}$ is an example of a **mixed fraction**

4. Add the following fractions.

   (a) $\frac{3}{5} + \frac{4}{5}$
   (b) $\frac{6}{7} + \frac{4}{7}$
   (c) $3\frac{3}{5} + 2\frac{4}{5}$
   (d) $2\frac{6}{7} + 3\frac{4}{7}$

   **Ans:**
   (a) $\frac{3}{5} + \frac{4}{5} = \frac{3+4}{5} = \frac{7}{5} = 1\frac{2}{5}$
   (b) $\frac{6}{7} + \frac{4}{7} = \frac{6+4}{7} = \frac{10}{7} = 1\frac{3}{7}$
   (c) $3\frac{3}{5} + 2\frac{4}{5} = \frac{18}{5} + \frac{14}{5} = \frac{32}{5} = 6\frac{2}{5}$
   (d) $2\frac{6}{7} + 3\frac{4}{7} = \frac{20}{7} + \frac{25}{7} = \frac{45}{7} = 6\frac{3}{7}$

---

**LESSON 17**

19. Solve

   $3x - 2y = 3$ \hspace{1cm} ...(1)
   $x + 4y = 8$ \hspace{1cm} ...(2)

   **Multiply each side Eq.1 by 2**

   $2(3x - 2y) = 2 \times 3$ \hspace{1cm} ...(1)
   $6x - 4y = 6$ \hspace{1cm} ...(1)
   $x + 4y = 8$ \hspace{1cm} ...(2)

   **Add each side of Eq.1 to Eq.2**

   $7x = 14$
   $x = \frac{14}{7} = 2$

   **Put this value of** $x = 2$ **into Eq.1 or Eq.2 to get** $y$

   $3x - 2y = 3 \Rightarrow 3 \times 2 - 2y = 3 \Rightarrow -2y = 3 - 6 = -3 \Rightarrow y = \frac{-3}{-2} = 1.5$
LESSON 18

PROBABILITIES

Example: Two boxes contain coloured bricks.
Box A contains 2 red bricks, 3 blue bricks and 1 yellow brick.
Box B contains 3 red bricks, 2 yellow bricks and 1 green brick.

Janet selects one brick from box A and one brick from box B.

Calculate the probability that the two bricks will be of the same colour.

Ans:

\[
P = \frac{2}{6} \times \frac{3}{6} + \frac{3}{6} \times \frac{0}{6} + \frac{1}{6} \times \frac{2}{6} + \frac{0}{6} \times \frac{1}{6} = \frac{6}{36} + 0 + \frac{2}{36} = \frac{8}{36} = \frac{2}{9}
\]

Example: Ahmad, Omar and Obaid are shooting at a target.
The probability of Ahmad hitting the target is 0.5
The probability of Omar hitting the target is 0.4
The probability of Obaid hitting the target is 0.2

If each of them fires one shot at the target, find the probability that:

(a) All three hit the target.
Ans: \(0.5 \times 0.4 \times 0.2 = 0.04\)

(b) None of them hit the target.
Ans: \((1 - 0.5) \times (1 - 0.4) \times (1 - 0.2) = 0.5 \times 0.6 \times 0.8 = 0.24\)

(c) At least one of them hit the target.
Ans: \(1 - 0.24 = 0.76\)
OR
Key: h = hit and m = miss

<p>| | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
<td>h h h</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>0</td>
<td>h h m</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>1</td>
<td>h m h</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>0</td>
<td>h m m</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>1</td>
<td>m h h</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>0</td>
<td>m h m</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>1</td>
<td>m m h</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>m m m</td>
</tr>
</tbody>
</table>

Or  At least one of them hit the target.

Ans: 

\[
0.04+0.16+0.06+0.24+0.04+0.16+0.06=0.76
\]

Example:  Bridget is given 11 coloured sweets:
3 of them green, 1 of them is black, and 7 of them are red.

(a)  (i) She eats one of the sweets. Assuming that she is equally fond
of each sort of sweet, what is the probability that the sweet
she eats is red?

Ans:

\[
\text{Red} \over \text{Total} = \frac{7}{3+1+7} = \frac{7}{11}
\]

(ii)  Bridget then eats a second sweet. What is the probability that
the first sweet is red and the second is black?

Ans: There will be 10 sweets remaining.

\[
\frac{7}{11} \times \frac{1}{10} = \frac{7}{110}
\]
(b) What is the probability that the first two sweets Bridget eats are both the same colour?

**Ans:**

\[
\left( \frac{7}{11} \times \frac{6}{10} \right) + \left( \frac{3}{11} \times \frac{2}{10} \right) = \frac{42}{110} + \frac{6}{110} = \frac{48}{110} = \frac{24}{55}
\]

**Example:** Imran plays a game of chess with his friend.

A game of chess can be *won* or *drawn* or *lost*.

The probability that Imran wins the game of chess is 0.3
The probability that Imran draws the game of chess is 0.25

Work out the probability that Imran *loses* the game of chess.

**Ans:***

\[1 - (0.3 + 0.25) = 1 - 0.55 = 0.45\]

**Example:** In a bag there are only red, blue and green balls.

(a) I am going to take a ball out of the bag at random.

Complete the table below.

<table>
<thead>
<tr>
<th>Colour of balls</th>
<th>Number of balls</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>Red</td>
<td>6</td>
<td></td>
</tr>
<tr>
<td>Blue</td>
<td></td>
<td>(\frac{1}{5})</td>
</tr>
<tr>
<td>Green</td>
<td>6</td>
<td></td>
</tr>
</tbody>
</table>

**Ans:**

\[
\left( \frac{x}{6 + 6 + x} \right) = \frac{1}{5}
\]

\[
\left( \frac{x}{12 + x} \right) = \frac{1}{5}
\]

\[5x = 12 + x\]
\[5x - x = 12\]
\[4x = 12\]
\[x = 3\]
(b) Before I take a ball out of the bag, I put one extra blue ball into the bag. What effect does this have on the probability that I will take a red ball?

<table>
<thead>
<tr>
<th>Colour of balls</th>
<th>Number of balls</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>Red</td>
<td>6</td>
<td>2/5</td>
</tr>
<tr>
<td>Blue</td>
<td>3</td>
<td>1/5</td>
</tr>
<tr>
<td>Green</td>
<td>6</td>
<td>2/5</td>
</tr>
</tbody>
</table>

Initial probability = \( \frac{2}{5} = \frac{16}{40} \)

Final probability = \( \frac{3}{8} = \frac{15}{40} \)

Therefore, probability decreases, i.e. \( \frac{15}{40} < \frac{16}{40} \)

Tick (✔) the correct box.
14. A bag contains counters that are red, black, or green.

- \(\frac{1}{3}\) of the counters are red
- \(\frac{1}{6}\) of the counters are black

There are 15 green counters in the bag.

How many black counters are in the bag?

Green ratio \(= 1 - \left(\frac{1}{3} + \frac{1}{6}\right) = 1 - \frac{3}{6} = \frac{1}{2}\)

Total counters \(= 2 \times 15 = 30\)

Total black counters \(= \frac{1}{6} \times 30 = 5\)
Example: A triangle has sides of length 28 cm, 39 cm and 21 cm.
(a) Calculate the size of the largest angle in the triangle.
(b) Calculate the size of the smallest angle in the triangle.

Ans:

\[
\sin B = \frac{28}{39}, \quad \sin A = \frac{21}{39}, \quad \sin C = \frac{21}{39}
\]

Because \(39 > 28 > 21\) Then, \(\sin B > \sin A > \sin C\) As a result: \(\hat{B} > \hat{A} > \hat{C}\)

Note: Both numerator and denominator decreases

\[
\frac{48}{96} = \frac{24}{48} = \frac{12}{24} = \frac{6}{12} = \ldots
\]

Note: Both numerator and denominator increases

\[
39^2 = 21^2 + 28^2 - 2 \times 21 \times 28 \times \cos B \Rightarrow \cos B = \frac{21^2 + 28^2 - 39^2}{2 \times 21 \times 28} = \frac{1225 - 1521}{1176} = -0.2517 \Rightarrow B = 104.58^\circ
\]

\[
21^2 = 28^2 + 39^2 - 2 \times 28 \times 39 \times \cos C \Rightarrow \cos C = \frac{28^2 + 39^2 - 21^2}{2 \times 28 \times 39} = \frac{2305 - 441}{2184} = 0.85358 \Rightarrow C = 31.41^\circ
\]

\[
28^2 = 21^2 + 39^2 - 2 \times 21 \times 39 \times \cos A \Rightarrow \cos A = \frac{21^2 + 39^2 - 28^2}{2 \times 21 \times 39} = \frac{1962 - 784}{1638} = 0.7192 \Rightarrow C = 44.01^\circ
\]

Example:
PQR and STUV are parallel straight lines.

(i) Work out the value of the angle marked $x^\circ$.

\[
\begin{align*}
\text{Ans: } & \quad x + 90 + 34 = 180 \\
& \quad x = 180 - 90 - 34 \\
& \quad x = 56^\circ
\end{align*}
\]

Example: The diagram shows three straight lines.

Work out the sizes of angles $a$, $b$ and $c$
Give reasons for your answers.

\[
\begin{align*}
\text{Ans: } & \quad b = 60^\circ \\
& \quad a = 180 - 130 = 50^\circ \\
& \quad c = 180 - (60 + 50) = 180 - 110 = 70^\circ
\end{align*}
\]
\[
\begin{align*}
\text{Angle } a \text{ is on a straight line with } 130, \text{ so } a &= 180 - 130 \\
a \text{ is supplementary with } 130, \text{ so } a + 130 &= 180 \\
\text{The angle vertically opposite } 130 \text{ is } 130, 360 - (130 + 130) = 100, \text{ (angles at a point)} \\
a \text{ is } \frac{100}{2} = 50 \text{ (also vertically opposite)}
\end{align*}
\]

\[
\begin{align*}
\text{Angle } b \text{ is vertically opposite the angle } 60^\circ, \text{ so it is also } 60^\circ \\
\text{The angle on a straight line with } b \text{ is } 120, \text{ so } b = 360 - 120 - 120 - 60 \text{ (angles at a point)}
\end{align*}
\]

\[
\begin{align*}
\text{There are } 180^\circ \text{ in a triangle, so } c &= 180 - 60 - 50 = 70^\circ \\
\text{The exterior angle of a triangle is equal to the sum of the two opposite interior angles, so } c &= 130 - 60
\end{align*}
\]

**Example:**
(a) Use \( \tan 35^\circ \) as 0.7 to work out length \( k \)

\[
\tan 35^\circ = \frac{k}{10} = 0.7 \quad \Rightarrow \quad k = 7 \text{ cm}
\]

\[
k = 7 \text{ cm}
\]

(b) Now use \( \tan 35^\circ \) as 0.7 to work out the area of this isosceles triangle.
You must show your working.

You must show your working.
\[ \tan 35^\circ = \frac{h}{10} = 0.7 \Rightarrow h = 7 \, cm \]
\[ \text{Area} = \frac{20 \times 7}{2} = \frac{140}{2} = 70 \, cm^2 \]

\[ \text{Area} = \frac{70}{7} \, cm^2 \]

Example: AC is the diameter of a circle and B is a point on the circumference of the circle.
What is the size of angle $x$?

\[ x = \square \]

Example: Write a number in each box to make the inequalities true.

\[
\begin{align*}
\square & \div \square < -1 \\
-1 & < \square \div \square < 0
\end{align*}
\]

Example: Two pupils each drew a triangle with one side of 5cm, one angle of $20^\circ$ and one angle of $60^\circ$. Must their triangles be congruent?

\[ \begin{array}{c}
\text{Yes} \\
\text{No}
\end{array} \]

Explain your answer.

9. Look at the cube. The area of a face of the cube is $9x^2$

\[
\text{Area} = 9x^2
\]

(a) Write an expression for the total surface area of the cube. Write your answer as simply as possible.

Ans:

Total surface area of the cube $= 6 \times 9x^2 = 54x^2$
(b) Write an expression for the volume of the cube.

Write your answer as simply as possible.
Ans:
side of the cube \(= \sqrt{9x^2} = 3x \)

Volume of the cube \(= (3x) \times (3x) \times (3x) = 27x^3 \)

Example: Look at the rectangle

The total area of the rectangle is \(30cm^2\)

Work out lengths \(x\) and \(y\)
Ans:

\[
Total\ Area = 4 \times \left(\frac{x}{2}\right) = \frac{4x}{2} = 2x = 30 \Rightarrow x = 15\ cm
\]

\[
2y + 6.1 = \frac{x}{2} = \frac{15}{2} = 7.5 \Rightarrow 2y = 7.5 - 6.1 = 1.42 \Rightarrow y = \frac{1.42}{2} = 0.71\ cm
\]

\(\Rightarrow x = 15\ cm\quad \text{and}\quad y = 0.71\ cm\)

Example: (a) Bags A and B contain some counters.
The number of counters in each bag is the same. Work out the value of $y$

**Ans:**
Counters in Bag A = Counters in Bag B

\[
\frac{1}{3} y + 4 = \frac{2}{3} y + 3
\]

\[
\frac{1}{3} y - \frac{2}{3} y = 3 - 4
\]

\[
- \frac{1}{3} y = -1
\]

\[
\Rightarrow y = 3
\]

(b) Bag C contains more counters than bag D.

What is the greatest possible value of $k$?

**Ans:**

\[
4k > 5k - 12
\]

\[
4k - 5k > -12
\]

\[
-k > -12
\]

\[
k < 12
\]

The greatest possible value of $k=11$

(c) Bag C contains less counters than bag D.

What is the smallest possible value of $k$?
Ans:

\[4k < 5k - 12\]
\[4k - 5k < -12\]
\[-k < -12\]
\[k > 12\]

The smallest possible value of \(k = 12\)

Example: Inside the rectangle below is a shaded rhombus. The vertices of the rhombus are the midpoints of the sides of the rectangle.

What is the area of the shaded rhombus?

Example: This shape is made of four congruent rectangles. Each rectangle has side lengths \(2x\) and \(x\)
The perimeter of the shape is 64cm. Work out the area of the shape.

Example: The diagram shows the net of a cube made of 6 squares.

K is the point (20, 10)
What are the coordinates of the points L and M?

L is \((-10, 20)\)

M is \((30, 20)\)

Example: The diagram shows a rhombus. The midpoints of two of its sides are joined with a straight line.
What is the size of angle $p$?

\[
a + a + 110 = 180 \Rightarrow 2a = 180 - 110 = 70 \Rightarrow a = 35^\circ \\
p = 180 - 35 = 145^\circ
\]

Example: A square of area $64 \, cm^2$ is cut to make two rectangles, A and B.

The ratio of area A to area B is $3 : 1$

Work out the dimensions of rectangles A and B.

\[
Area \, A = \frac{64}{3+1} \times 3 = 48 \, cm^2
\]
Area \( B = \frac{64}{3+1} \times 1 = 16\ cm^2 \)

**Example:** A pole is held vertical by two sloping wires, one of length 12 m. The wires are fixed from the top of the pole to two different points on the ground, as shown.

How long is the second wire?

**Ans:**

\[
p = \sqrt{(12)^2 - (6.5)^2} = \sqrt{144 - 42.25} = \sqrt{101.75} = 10.087
\]

\[
w = \sqrt{(10.087)^2 + (9.6)^2} = \sqrt{101.75 + 92.16} = \sqrt{193.91} = 13.925 = 13.9\ (1\ dp)
\]

**Note:** Do not round off intermediate result such as the value of \( h \) in this case as the final value is \( w \). Round off \( w \) at the end.

The length of each side of an equilateral triangle is \((x+5)\) centimetres.
(a) Find an expression, in terms of $x$, for the perimeter of the equilateral triangle.

**Ans:** $P = 3(x + 5) = 3x + 15$ \hspace{1cm} (2 Marks)

The perimeter of the equilateral triangle is 22.5 cm.

(b) Work out the value of $x$. \hspace{1cm} (5 Marks)

**Ans:**

\[
3x + 15 = 22.5 \\
3x = 22.5 - 15 \\
3x = 7.5 \\
x = \frac{7.5}{3} = 2.5
\]

Example: Town B is 4.5 km due West of town C.
Town A is 2.4 km due North of town B.

(a) Calculate the size of the angle marked $x$.
Give your answer correct to 3 significant figures.

**Ans:**

\[
\tan x = \frac{2.4}{4.5} = 0.5333 \\
x = \tan^{-1}(0.5333) = 28.07^0
\]

(b) Find the bearing of town C from town A.
Give your answer correct to 3 significant figures.
\[ \begin{align*}
  y &= 180 - (28.07 + 90) = 61.93^\circ \\
  z &= 180 - 61.93^\circ = 118.07^\circ 
\end{align*} \]

**Example:**

\[ \begin{align*}
  \triangle ABC \text{ is a triangle.} \\
  AB &= 9 \text{ cm} \\
  BC &= 15 \text{ cm} \\
  \text{Angle } \angle ABC &= 110^\circ 
\end{align*} \]

Calculate the area of the triangle.
Give your answer correct to 3 significant figures.

**Ans:**

\[ \begin{align*}
  A &= \frac{1}{2} \times 9 \times 15 \times \sin(110^\circ) \\
  &= 67.5 \times 0.9396926 \\
  &\approx 63.4 \text{ cm}^2 
\end{align*} \]

5 The triangle XYZ is shown below with

\[ \begin{align*}
  XY &= 18 \text{ cm}, \\
  YZ &= 25 \text{ cm}, \\
  ZX &= 21 \text{ cm}. 
\end{align*} \]
Calculate the size of:

(a) Angle X
Ans:
Cosine Rule:  
\[ 25^2 = 18^2 + 21^2 - 2 \times 18 \times 21 \times \cos X \]
\[ \cos X = \frac{18^2 + 21^2 - 25^2}{2 \times 18 \times 21} = \frac{140}{756} = 0.185 \]
\[ X = \cos^{-1}(0.185) = 79.3^0 \]

(b) Angle Y
Ans:
\[ 21^2 = 18^2 + 25^2 - 2 \times 25 \times 18 \times \cos Y \]
\[ \cos Y = \frac{18^2 + 25^2 - 21^2}{2 \times 25 \times 18} = \frac{508}{900} = 0.5644 \]
\[ Y = \cos^{-1}(0.5644) = 55.6^0 \]

(c) Calculate the area of the triangle.
Ans:
\[ A = \frac{1}{2} (18 \times 25) \sin Y = \frac{1}{2} (18 \times 25) \sin(55.6^0) = 225 \times 0.825 = 185.65 \text{ cm}^2 \]
OR
\[ A = \frac{1}{2} (18 \times 21) \sin X = \frac{1}{2} (18 \times 21) \sin(79.3^0) = 189 \times 0.9826 = 185.71 \text{ cm}^2 \]

Example: Look at triangle ABC. ABD is an isosceles triangle where AB = AD.
Work out the sizes of angles $x$, $y$ and $z$

Give reasons for your answers.

$x = 74\degree$  
Because $ABD$ is an isosceles triangle in which $AB = AD$ 
And therefore the TWO angles are equal

$y = 32\degree$  
Because $y = 180 - (74 + 74) = 32$ degrees 
The sum of the angles of the triangle $= 180$ degrees

$z = 46\degree$  
Because $z = 180 - (74 + 32 + 28) = 46$ degrees 
The sum of the angles of the triangle $= 180$ degrees

Example: (a) The graphs show information about the different journeys of four people.
Write the correct names next to the journey descriptions in the table below.

**Ans:**

<table>
<thead>
<tr>
<th>Name</th>
<th>Journey description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Chris</td>
<td>This person walked slowly and then ran at a constant speed.</td>
</tr>
<tr>
<td>Dee</td>
<td>This person walked at a constant speed but turned back for a while before continuing.</td>
</tr>
<tr>
<td>Ann</td>
<td>This person walked at a constant speed without stopping or turning back.</td>
</tr>
<tr>
<td>Ben</td>
<td>This person walked at a constant speed but stopped for a while in the middle.</td>
</tr>
</tbody>
</table>

(b) Ella made a different journey of 4km. She walked to a place 4km away from her starting point. Here is the description of her journey.
For the first 15 minutes she walked at 4km per hour.
For the next 15 minutes she walked at 2km per hour.
For the last 30 minutes she walked at a constant speed.

Show Ella’s journey accurately on the graph below.

**Ans:**

\[
\text{Speed} = \frac{4 \text{ km}}{60 \text{ min}} = \frac{4 \text{ km}}{4 \text{ min}} = \frac{1 \text{ km}}{15 \text{ min}}
\]

\[
\text{Speed} = \frac{2 \text{ km}}{60 \text{ min}} = \frac{2 \text{ km}}{4 \text{ min}} = \frac{0.5 \text{ km}}{15 \text{ min}}
\]

\[
\text{Speed} = \frac{4 - 1.5 \text{ km}}{30 \text{ min}} = \frac{2.5 \text{ km}}{30 \text{ min}}
\]
because, she needs to travel the remaining 2.5 km in 30 minutes with a constant speed (straight line)
(c) For the last 30 minutes of her journey, what was Ella’s speed?

\[
\text{Speed} = \frac{4 - 1.5 \text{ km}}{30 \text{ min}} = \frac{2.5 \text{ km}}{30 \text{ min}} = \frac{2.5 \text{ km}}{30 \text{ min}} \times \frac{hr}{60 \text{ min}} = \frac{5 \text{ km}}{hr}
\]

5 Km per hour

Example: Kaylee has some 1cm cubes.
She makes a solid cube with side length 6cm out of the cubes.

Then she uses all these cubes to make some cubes with side length 2cm.
How many of these 2cm cubes can Kaylee make?

Ans:
Total volume = \(6cm \times 6cm \times 6cm = 216 cm^3\)
Volume of a cube 2cm = \(6cm \times 6cm \times 6cm = 216 cm^3\)
Number of 2cm cube required = \(\frac{216 cm^3}{8 cm^3} = 27\)

27

Example: (a) Look at this triangle. Work out length AC.
Ans:

\[ AC^2 = 15^2 - 8^2 = 225 - 64 = 161 \]

\[ AC = \sqrt{161} = 12.689 \text{ cm} \]

(b) Look at this triangle. Work out length DF.

\[ \tan 37^0 = \frac{DF}{13} \]

\[ DF = 13 \times \tan 37^0 = 13 \times 0.75355 = 9.7962 \approx 9.8 \text{ cm} \]

Example: The graph shows a circle with centre \((0, 0)\). The circle has the equation:

\[ x^2 + y^2 = 25 \]
(a) There are two points on the circumference of the circle with an $x$-coordinate of 4.

Complete the coordinates of these two points.

**Ans:**

\[ x^2 + y^2 = 25 \]
\[ 4^2 + y^2 = 25 \]
\[ 16 + y^2 = 25 \]
\[ y^2 = 25 - 16 \]
\[ y^2 = 9 \]
\[ y = \sqrt{9} = \pm 3 \]

\( (4, 3) \) and \( (4, -3) \)

(b) What is the radius of the circle?

\[ x^2 + y^2 = r^2 \]
\[ r^2 = 25 \]
\[ r = \sqrt{25} = 5 \]

\( 5 \)

(c) Point P is on the circumference of the circle. Its $x$-coordinate is equal to its $y$-coordinate.

What are the coordinates of point P, correct to 1 decimal place?

\[ x^2 + y^2 = 25 \]
\[ x = y \]

\[ x^2 + x^2 = 2x^2 = 25 \]
\[ x^2 = \frac{25}{2} \]
\[ x = \sqrt{\frac{25}{2}} = 3.535 \approx 3.5 \]
\[ y = x \approx 3.5 \]

\( P \) is \( (3.5, 3.5) \)
Example: A cube is cut through four of its vertices, A, B, C and D, into two identical pieces. The diagram below shows one of the pieces.

Find the length of the line AC.

\[ BC = \sqrt{4^2 + 4^2} = \sqrt{16 + 16} = \sqrt{32} \]
\[ AC = \sqrt{4^2 + \left(\sqrt{32}\right)^2} = \sqrt{16 + 32} = \sqrt{48} = 6.928 \approx 6.93 \text{ cm} \]

\[ AC = 6.93 \text{ cm} \]

Example: A window is made with two pieces of glass. One piece is a square, the other is a semicircle.

The area of the square is \(1m^2\)
What is the area of the semicircle?  
Give your answer in $cm^2$ to the nearest whole number.

\[
\text{Area of square} = a \times a = a^2 = 1m^2 \Rightarrow a = 1m
\]

\[
\text{Area of the semicircle} = \pi \times r^2 = \pi \times \left(\frac{1}{2}\right)^2 = \frac{\pi}{4} = 0.7854 \ m^2 = 0.7854 \times 10000 = 7854 \ cm^2
\]

\[
\frac{7458}{cm^2}
\]

Example: The quadrilateral shown has angles of $x$, $79^0$, $3x$ and $97^0$.  
Work out the value of $x$?

\[
\begin{align*}
\text{(Diagram not drawn accurately)} \\
\end{align*}
\]

Ans:  
\[
\begin{align*}
79 + 3x + 97 + x &= 360 \\
4x + 176 &= 360 \\
4x &= 360 - 176 \\
4x &= 184 \\
x &= \frac{184}{4} = 46
\end{align*}
\]

Example: In the triangle ABC shown above, BC = 8.5 cm and AX = 6.4 cm. Calculate the area of the triangle ABC.
Example: ABCDE is a regular pentagon, with O as its centre. Calculate the size of the angle AOB.

\[ \text{Ans: } AOB = \frac{360}{5} = 72^\circ \]

Example: OPQ is a sector of the circle, centre O, with radius 9cm. Angle POQ = 45°

(a) Work out the perimeter of the sector OPQ

\[ \text{Ans: } \text{Perimeter} = 9 + 9 + \text{arc of } PQ \]

\[ \text{arc of } PQ = r\theta, \text{ where } \theta \text{ is in radian} \]

\[ \text{arc of } PQ = 9 \times \frac{45}{180} \times \pi = 7.0686 \text{ cm} \]

\[ \text{Perimeter} = 9 + 9 + 7.0686 = 25.0686 \approx 25.07 \text{ cm} \]
(b) Work out the area of the sector OPQ

**Ans:** $\text{Area of sector } OPQ = \frac{1}{2} r^2 \theta$, where $\theta$ is in radian

$\text{Area of sector } OPQ = \frac{1}{2} \times 9^2 \times \left( \frac{45}{180} \times \pi \right) = 31.8 \text{ cm}^2$

**Example:**
(a) A regular polygon has an exterior angle of $18^0$
Find the number of sides in the polygon.

(b) The diagram shows a play tent in the shape of a triangular prism.
Calculate the volume of the tent.
Show all your working.

![Diagram of a play tent](image)

80 cm
100 cm
120 cm

**Example:**
(a) The diagram shows a ladder, AC, leaning against the wall of a house.

![Diagram of a ladder](image)

In the triangle ABC, angle ABC = $90^0$, angle CAB = $53^0$ and CB = 85 cm.
How high up the wall does the ladder reach?

**Ans:**

$$\frac{85}{\sin 53^0} = \frac{AB}{\sin 37^0},$$

$$\Rightarrow AB = \frac{85 \times \sin 37^0}{\sin 53^0} = \frac{85 \times 0.601815}{0.7986355} = 64.05 \text{ cm}$$
In the diagram above TQ is parallel to SR. Triangles PQT and PRS are similar. 
PT = 3.1 cm, TS = 4.4 cm. RS = 5 cm
Find the length of QT, giving your answer correct to 3 significant figures.
\[\text{Ans: } QT = \frac{3.1}{3.1 + 4.4} = \frac{3.1}{7.5} = 0.4133 = 0.433 \text{ (3 s.f) },\]

Example: The diagram represents a regular heptagon with two of its lines of symmetry shown.

(a) Write down the value of angle \(x\).
\[\text{Ans: } x = 90^0\]

(b) Calculate the values of the following angles.
(i) Angle \(w\).
\[\text{Ans: } w = \frac{1}{2} \left( \frac{360}{7} \right) = \frac{360}{14} = 25.7^0\]

(ii) Angle \(y\).
\[\text{Ans: } z = 2 \times w = 2 \times 25.7 = 51.4^0\]
\[y = 180 - 51.4 = 128.6^0\]
Example: The diagram below shows part of a basketball court known as ‘the key’. The key consists of a trapezium and a semi-circle. The dimensions are shown on the diagram, which is not drawn to scale.

What is the total area of ‘the key’?

Ans:

Area of the trapezium = \( \frac{1}{2} (1.4 + 2.4) \times 5.2 = \frac{19.76}{2} = 9.88 \text{ m}^2 \)

Area of the semi-circle = \( \frac{1}{2} \times \pi \left( \frac{1.4}{2} \right)^2 = \frac{1.96 \times \pi}{8} = 0.7696 \text{ m}^2 \)

Total area = 9.88 + 0.7696 = 10.6496 \( \approx \) 10.6 \text{ m}^2

Example: A rectangle has a length of \((x + 4)\) cm and a width of \((x - 1)\) cm.

(a) If the perimeter of the rectangle is 40 cm, what is the value of \(x\)?

Ans:

Perimeter = \((x + 4) + (x + 4) + (x - 1) + (x - 1) = 40\)

\(4x + 4 + 4 - 1 - 1 = 40\)

\(4x = 40 + 2 - 8\)
4x = 34
x = \frac{34}{4} = 8.5

(b ) (i) If the area of the rectangle is \(84 \text{ cm}^2\), show that \(x^2 + 3x - 88 = 0\)

\text{Ans:}
\begin{align*}
\text{Area} &= (x + 4)(x - 1) = 84 \\
x^2 - x + 4x - 4 &= 84 \\
x^2 + 3x &= 84 + 4 \\
x^2 + 3x &= 88 \\
x^2 + 3x - 88 &= 0
\end{align*}

(ii) Find the value of \(x\) when the area of the rectangle is \(84 \text{ cm}^2\).

\text{Ans:}
\begin{align*}
x^2 + 3x - 88 &= 0 \\
(x + 11)(x - 8) &= 0
\end{align*}

If \((x + 11) = 0\) \(\Rightarrow x = -11\) Not OK
If \((x - 8) = 0\) \(\Rightarrow x = 8\) OK

6 The triangle XYZ is shown below with
\(XY = 18 \text{ cm}, \ YZ = 25 \text{ cm}, \ ZX = 21 \text{ cm}\).

![Diagram of triangle XYZ]

Calculate the size of:

(a) Angle \(X\)

\text{Ans:}
\begin{align*}
\text{Cosine Rule:} \quad 25^2 &= 18^2 + 21^2 - 2 \times 18 \times 21 \times \cos X \\
\cos X &= \frac{18^2 + 21^2 - 25^2}{2 \times 18 \times 21} = \frac{140}{756} = 0.185 \\
X &= \cos^{-1}(0.185) = 79.3^0
\end{align*}

(b) Angle \(Y\)
Ans:

\[ 21^2 = 18^2 + 25^2 - 2 \times 25 \times 18 \cos Y \]
\[ \cos Y = \frac{18^2 + 25^2 - 21^2}{2 \times 25 \times 18} = \frac{508}{900} = 0.5644 \]
\[ Y = \cos^{-1}(0.5644) = 55.6^0 \]

(c) Calculate the area of the triangle.

Ans:

\[ A = \frac{1}{2} (18 \times 25) \sin Y = \frac{1}{2} (18 \times 25) \sin(55.6^0) = 225 \times 0.825 = 185.65 \text{ cm}^2 \]

OR

\[ A = \frac{1}{2} (18 \times 21) \sin X = \frac{1}{2} (18 \times 21) \sin(79.3^0) = 189 \times 0.9826 = 185.71 \text{ cm}^2 \]

Example: A, B, C, D and E are points on the circumference of the circle, centre O.

Angle AOD = 126\(^0\).

Calculate the size of:

(a) Angle ABD

Ans:

\[ \text{Angle ABD} = \frac{1}{2} AOD = \frac{126}{2} = 63^0 \]

(a) Angle AED
Ans:

Angle $ABD = \frac{1}{2} AOD = \frac{126}{2} = 63^0$

(b) Angle $ACD$

Example: The diagram shows sand stored in a conical hopper. The radius of the top of the cone is 2 m, and the depth of the cone is 5 m.

When the hopper is full, the sand forms a conical pile of height 1.5 m at the top of the hopper. Calculate to suitable degrees of accuracy:

(a) The total volume of sand held by the hopper when it is full

(a) The curved surface area of the conical hopper.
LESSON 20

Rational and Irrational Numbers

Rational Numbers
Most of the numbers we use in everyday life are rational numbers.
A rational number is any number that can be expressed as a vulgar fraction, i.e.
written in the form \( \frac{a}{b} \), where \( a \) and \( b \) are whole numbers.
All integers (positive and negative whole numbers) are rational because they can be written as \( \frac{a}{1} \).
All vulgar fractions are rational because they are already in the form \( \frac{a}{b} \).
All terminating decimals (decimals which stop after a number of places) are rational because they can be written in the form \( \frac{a}{10^n} \).

E.g. 0.375 stops after 3 decimal places so it can be written as \( \frac{375}{1000} \).

All recurring decimals (decimals which repeat after a number of places) are rational.
(Remember dots are used to indicate the first and last number in any repeating pattern.)

E.g. \( 0.\overline{54} = 0.545454 \ldots \) repeats after two decimal places.

\[
\begin{align*}
x &= 0.\overline{54} & \text{(1)} \\
x &= 0.\overline{54} = 0.54\overline{54} \\
100x &= 54.\overline{54} & \text{(2)}
\end{align*}
\]

Subtracting Eq.1 from 2:
\[
100x - x = 54.\overline{54} - 0.54
\]
\[
99x = 54
\]
\[
x = \frac{54}{99} = \frac{6}{11}
\]

Note: All rational numbers can be expressed as decimals.
Irrational Numbers

An irrational number is one which is not rational. An irrational number cannot be expressed as \( \frac{a}{b} \) with \( a \) and \( b \) both whole numbers. If you expressed an irrational number as a decimal it would go on for ever (to infinity) without repeating.

There are many more irrational numbers than rational numbers but you will mainly use rational numbers. However, there are two irrational numbers you will use often. These are \( \pi \) and \( \sqrt{2} \).
If you look at each of these values on your calculator you will see that they do not repeat.

Note: An irrational number multiply by an irrational number may become rational. An irrational number divided by an irrational number may become rational.

Example: Express 0.\( \dot{3} \) as a fraction

\[
x = 0.\dot{3} 
\]
\[
x = 0.3 = 0.3\dot{3} 
\]
\[
10x = 3.\dot{3} 
\]

Subtracting Eq.1 from 2:

\[
x = 0.3 = 0.3\dot{3} 
\]
\[
10x = 3.\dot{3} 
\]
\[
10x - x = 3.\dot{3} - 0.\dot{3} 
\]
\[
9x = 3 
\]
\[
x = \frac{3}{9} = \frac{1}{3} 
\]

Example: Express 0.\( \dot{3}\dot{3} \) as a fraction

\[
x = 0.\dot{3}\dot{3} 
\]
\[
x = 0.3\dot{3} = 0.3\dot{3} \dot{3} 
\]
\[
100x = 33.\dot{3}\dot{3} 
\]
Subtracting Eq.1 from 2:
\[100x - x = 33.3\overline{3} - 0.3\overline{3}\]
\[99x = 33\]
\[x = \frac{33}{99} = \frac{1}{3}\]

**Example:** Express 0.3\overline{4} as a fraction
\[x = 0.3\overline{4} \quad \text{...(1)}\]
\[x = 0.34 = 0.3434\]
\[100x = 34.34 \quad \text{...(2)}\]

Subtracting Eq.1 from 2:
\[100x - x = 34.3\overline{4} - 0.3\overline{4}\]
\[99x = 34\]
\[x = \frac{34}{99}\]

**Example:** Express 0.1\overline{3} as a fraction
\[x = 0.1\overline{3} \quad \text{...(1)}\]
\[10x = 1.\overline{3} = 1.33 \quad \text{...(1)} \quad \textbf{Note:} \text{ Very important!}\]
\[\textbf{N.B.:} \quad \text{Move the decimal point to the occurring number correctly}\]
\[10x = 1.\overline{3} = 1.33 \quad \text{...(1)}\]
\[100x = 13.3 \quad \text{...(2)}\]

Subtracting Eq.1 from 2:
\[100x - 10x = 13.3\overline{3} - 1.3 = 12\]
\[90x = 12\]
\[x = \frac{12}{90} = \frac{2}{15}\]

**Example:** Express 0.5\overline{90} as a fraction
\[x = 0.5\overline{90} \quad \text{...(1)}\]
\[10x = 5.\overline{90} \quad \text{...(1)} \quad \textbf{Note:} \text{ Very important!}\]
\[\textbf{N.B.:} \quad \text{Move the decimal point to the occurring number correctly}\]
\[10x = 5.\overline{90} \quad \text{...(1)}\]
\[1000x = 590.\overline{90} \quad \text{...(2)}\]
Subtracting Eq.1 from 2:

\[ 1000x - 10x = 590.90 - 5.90 = 585 \]
\[ 990x = 585 \]
\[ x = \frac{585}{990} \]

Example: Express \( 0.3\overline{4}8 \) as a fraction

\[ x = 0.3\overline{4}8 \quad ... (1) \]
\[ 100x = 34.\overline{8} \quad ... (1) \quad \text{Note: Very important!} \]

N.B: Move the decimal point to the occurring number correctly

\[ 100x = 34.\overline{8} \quad ... (1) \]
\[ 1000x = 348.\overline{8} \quad ... (2) \]

Subtracting Eq.1 from 2:

\[ 1000x - 100x = 348.\overline{8} - 34.\overline{8} = 314 \]
\[ 900x = 314 \]
\[ x = \frac{314}{900} = \frac{157}{450} = 0.348\overline{8} \quad ⇒ OK \]

Example: Express \( 0.7\overline{8}6 \) as a fraction

\[ x = 0.7\overline{8}6 \quad ... (1) \]
\[ 1000x = 786.\overline{7}86 \quad ... (2) \]

Subtracting Eq.1 from 2:

\[ 1000x - x = 786 \]
\[ 999x = 786 \]
\[ x = \frac{786}{999} = \frac{262}{333} = 0.786\overline{786} \quad ⇒ OK \]

Example: Express \( 0.4\overline{0}27 \) as a fraction

\[ 0.4\overline{0}27 = 0.4027027027 \quad ... \]
\[ x = 0.4\overline{0}27 = 0.4027027027 \quad ... \]
\[ 10x = 4.\overline{0}27 \quad ... (1) \]
\[ 10000x = 4027.\overline{0}27 \quad ... (2) \]
Subtracting Eq.1 from 2:
\[ 10000x - 10x = 4023 \]
\[ 9990x = 4023 \]
\[ x = \frac{4023}{9990} = 0.4027027027...027 \Rightarrow OK \]

**Example:** Express 0.0237 as a fraction

\[ x = 0.0237 \ldots (1) \]
\[ 100x = 2.37 \ldots (1) \quad \textbf{Note: Very important!} \]

\textbf{N.B.:} Move the decimal point to the occurring number correctly

\[ 100x = 2.37 = 2.3737 \ldots (1) \]
\[ 10000x = 237.37 \ldots (2) \]

Subtracting Eq.1 from 2:
\[ 10000x - 100x = 237.37 - 2.37 = 235 \]
\[ 9900x = 235 \]
\[ x = \frac{235}{9900} \]

**Example:** Express -0.0237 as a fraction

\[ x = -0.0237 \ldots (1) \]
\[ 100x = -2.37 \ldots (1) \quad \textbf{Note: Very important!} \]

\textbf{N.B.:} Move the decimal point to the occurring number correctly

\[ 100x = -2.37 = -2.3737 \ldots (1) \]
\[ 10000x = -237.37 \ldots (2) \]

Subtracting Eq.1 from 2:
\[ 10000x - 100x = -237.37 - \left( -2.37 \right) = -237.37 + 2.37 = -235 \]
\[ 9900x = -235 \]
\[ x = \frac{-235}{9900} = -\frac{47}{1980} \]

**Example:** Express 9.02538 as a fraction

\[ x = 9.02538 \ldots (1) \]
\[ 1000x = 9025.38 \ldots (1) \quad \textbf{Note: Very important!} \]
N.B: Move the decimal point to the occurring number correctly

\[ 1000 \, x = 9025.38 \quad \ldots(1) \]
\[ 100000 \, x = 902538.38 \quad \ldots(2) \]

Subtracting Eq. 1 from 2:
\[ 100000 \, x - 1000 \, x = 902538.38 - 9025.38 = 893513 \]
\[ 99000 \, x = 893513 \]
\[ x = \frac{893513}{99000} = 9.025383838 \ldots38 \Rightarrow OK \]

Example: Express 0.15 as a fraction
\[ x = 0.15 \quad \ldots(1) \]
\[ 10 \, x = 1.5 \quad \ldots(1) \quad \text{Note: Very important!} \]
N.B: Move the decimal point to the occurring number correctly

\[ 10 \, x = 1.5 \quad \ldots(1) \]
\[ 100 \, x = 15.5 \quad \ldots(2) \]

Subtracting Eq. 1 from 2:
\[ 100 \, x - 10 \, x = 14 \]
\[ 90 \, x = 14 \]
\[ x = \frac{14}{90} = \frac{7}{45} = 0.155555 \ldots5 \Rightarrow OK \]

Example: Express 14.23 as a fraction
\[ x = 14.23 \quad \ldots(1) \]
\[ 100 \, x = 1423.23 \quad \ldots(1) \quad \text{Note: Very important!} \]
N.B: Move the decimal point to the occurring number correctly

\[ 100 \, x = 1423.23 \quad \ldots(1) \]
\[ x = 14.23 \quad \ldots(2) \]
Subtracting Eq.1 from 2:
\[100x - x = 1423.23 - 14.23 = 1409\]
\[99x = 1409\]
\[x = \frac{1409}{99} = 14.23232323 \ldots 23 \Rightarrow OK\]

Example: Express \(0.142857\ldots\) as a fraction

\[x = 0.142857 \ldots \] \((1)\)

Subtracting Eq.1 from 2:
\[1000000x - x = 142857 \ldots \]
\[999999x = 142857 \ldots \] \((1)\)
\[x = \frac{142857}{999999} = \frac{1}{7} = 0.142857 \ldots \Rightarrow OK\]

Example: Write a fraction which is equivalent to the recurring decimal \(0.253\ldots\)

\[x = 0.253 \ldots \] \((1)\)

Subtracting Eq.1 from 2:
\[1000x - x = 253 \ldots \]
\[999x = 253 \ldots \] \((1)\)
\[x = 0.253 \ldots \] \((2)\)
\[ x = \frac{253}{999} = 0.253253 \ldots 253 \Rightarrow OK \]

**Vectors Analysis**

**Vector representation**

A vector can be represented in several ways:

(a) Geometrically (see Fig.1)

(b) As a column vector: 
\[
\vec{P_2P_1} = \begin{pmatrix} x_2 - x_1 \\ y_2 - y_1 \end{pmatrix} = \begin{pmatrix} 16 - 7 \\ 13 - 5 \end{pmatrix} = \begin{pmatrix} 9 \\ 8 \end{pmatrix}
\]
with the x-value on top of the y-value, and 
\[
\vec{P_2P_1} = \begin{pmatrix} 7 - 16 \\ 5 - 13 \end{pmatrix} = \begin{pmatrix} -9 \\ -8 \end{pmatrix}
\]

(c) In unit base vectors: The unit vector \( \begin{pmatrix} 1 \\ 0 \end{pmatrix} \) is a unit vector along the x-axis

and the unit vector \( \begin{pmatrix} 0 \\ 1 \end{pmatrix} \) is a unit vector along the y-axis. These vectors \( \begin{pmatrix} 1 \\ 0 \end{pmatrix} \)
and \( \begin{pmatrix} 0 \\ 1 \end{pmatrix} \) are called the unit base vectors. \( \begin{pmatrix} 1 \\ 0 \end{pmatrix} \) is given the letter \( i \) and \( \begin{pmatrix} 0 \\ 1 \end{pmatrix} \) is given the letter \( j \). The vector \( \begin{pmatrix} 9 \\ 8 \end{pmatrix} = 9i + 8j \)

Example: Find the displacement vector \( \overrightarrow{PQ} \) and write each vector:

(a) as a column vector (b) in terms of the unit base vectors.

(a) As a column vector: \( \overrightarrow{PQ} = \begin{pmatrix} q_x - p_x \\ q_y - p_y \end{pmatrix} = \begin{pmatrix} 15 - 6 \\ 6 - 13 \end{pmatrix} = \begin{pmatrix} 9 \\ -7 \end{pmatrix} \)

(b) In terms of \( i \) and \( j \): \( \overrightarrow{PQ} = 9i - 7j \)

Example: Points A, B, C, and D are indicated as follows:

Write as column vectors: \( \overrightarrow{AB} \), \( \overrightarrow{AD} \), \( \overrightarrow{CB} \), \( \overrightarrow{AC} \), \( \overrightarrow{DB} \).
$\vec{AB} = \begin{pmatrix} B_x - A_x \\ B_y - A_y \end{pmatrix} = \begin{pmatrix} 7 - 2 \\ 8 - 2 \end{pmatrix} = \begin{pmatrix} 5 \\ 6 \end{pmatrix}$. In terms of $i$ and $j$: $\vec{AB} = 5i + 6j$

$\vec{AD} = \begin{pmatrix} D_x - A_x \\ D_y - A_y \end{pmatrix} = \begin{pmatrix} 15 - 2 \\ 5 - 2 \end{pmatrix} = \begin{pmatrix} 13 \\ 3 \end{pmatrix}$. In terms of $i$ and $j$: $\vec{AD} = 13i + 3j$

$\vec{CB} = \begin{pmatrix} B_x - C_x \\ B_y - C_y \end{pmatrix} = \begin{pmatrix} 7 - 18 \\ 8 - 12 \end{pmatrix} = \begin{pmatrix} -11 \\ -4 \end{pmatrix}$. In terms of $i$ and $j$: $\vec{CB} = -11i - 4j$

**The modulus (magnitude) of a vector**

If $\vec{AB} = \begin{pmatrix} x \\ y \end{pmatrix} = xi + yj$, the $|\vec{AB}| = \sqrt{x^2 + y^2}$

If $\vec{AB} = \begin{pmatrix} 3 \\ 4 \end{pmatrix} = 3i + 4j$, the $|\vec{AB}| = \sqrt{3^2 + 4^2} = \sqrt{9 + 16} = \sqrt{25} = 5$

If $\vec{AB} = \begin{pmatrix} 3 \\ -4 \end{pmatrix} = 3i - 4j$, the $|\vec{AB}| = \sqrt{3^2 + (-4)^2} = \sqrt{9 + 16} = \sqrt{25} = 5$

If $\vec{AB} = \begin{pmatrix} -3 \\ -4 \end{pmatrix} = -3i - 4j$, then $|\vec{AB}| = \sqrt{(-3)^2 + (-4)^2} = \sqrt{9 + 16} = \sqrt{25} = 5$
Example: Calculate the modulus of the vectors: (a) \( \begin{pmatrix} 12 \\ 5 \end{pmatrix} \)  (b) \( \begin{pmatrix} -8 \\ 15 \end{pmatrix} \)  (c) \( 3i - 4j \)

(a) Modulus of \( \begin{pmatrix} 12 \\ 5 \end{pmatrix} \) is \( \sqrt{12^2 + 5^2} = \sqrt{144 + 25} = \sqrt{169} = 13 \)

(b) Modulus of \( \begin{pmatrix} -8 \\ 15 \end{pmatrix} \) is \( \sqrt{(-8)^2 + 15^2} = \sqrt{64 + 225} = \sqrt{289} = 17 \)

(c) Modulus of \( 3i - 4j \) is \( \sqrt{3^2 + (-4)^2} = \sqrt{9 + 16} = \sqrt{25} = 5 \)

Example: Consider the points: \( A(-1, 1) \)  \( B(2, 3) \)  \( C(4, 0) \)  \( D(1, -2) \)
State which pairs of vectors are equal in magnitude, parallel or both.

\[ \vec{BC} = \begin{pmatrix} C_x - B_x \\ C_y - B_y \end{pmatrix} = \begin{pmatrix} 4 - 2 \\ 0 - 3 \end{pmatrix} = \begin{pmatrix} 2 \\ -3 \end{pmatrix} \]
\[ \vec{BD} = \begin{pmatrix} D_x - B_x \\ D_y - B_y \end{pmatrix} = \begin{pmatrix} 1 - 2 \\ -2 - 3 \end{pmatrix} = \begin{pmatrix} -1 \\ -5 \end{pmatrix} \]
\[ \vec{AC} = \begin{pmatrix} C_x - A_x \\ C_y - A_y \end{pmatrix} = \begin{pmatrix} 4 - (-1) \\ 0 - 1 \end{pmatrix} = \begin{pmatrix} 5 \\ -1 \end{pmatrix} \]
\[ \vec{DA} = \begin{pmatrix} A_x - D_x \\ A_y - D_y \end{pmatrix} = \begin{pmatrix} -1 - 1 \\ 1 - (-2) \end{pmatrix} = \begin{pmatrix} -2 \\ 3 \end{pmatrix} \]

\[ \vec{BC} = -\vec{DA} \] or \( \vec{BC} = \vec{AD} \), then \( \vec{BC} \) and \( \vec{AD} \) are both equal in magnitude and parallel and \( |\vec{BD}| = |\vec{AC}| \)

Example: If \( \vec{A} = \begin{pmatrix} 5 \\ -1 \end{pmatrix} \), calculate: (a) \( 4\vec{A} \)  (b) \( -\vec{A} \)  (c) \( |\vec{A}| \)  (d) \( 3|\vec{A}| \)

(a) \( 4\vec{A} = 4\begin{pmatrix} 5 \\ -1 \end{pmatrix} = \begin{pmatrix} 20 \\ -4 \end{pmatrix} \)  (b) \( -\vec{A} = \begin{pmatrix} -5 \\ 1 \end{pmatrix} \)

(c) \( |\vec{A}| = \sqrt{(5)^2 + (-1)^2} = \sqrt{25 + 1} = \sqrt{26} \)  (d) \( 3|\vec{A}| = 3\sqrt{(5)^2 + (-1)^2} = 3\sqrt{25 + 1} = 3\sqrt{26} \)

Addition and subtraction of vectors

Vectors are added by adding the corresponding x- and y-coordinates.

\[ \begin{pmatrix} x_1 \\ y_1 \end{pmatrix} + \begin{pmatrix} x_2 \\ y_2 \end{pmatrix} = \begin{pmatrix} x_1 + x_2 \\ y_1 + y_2 \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} x_1 \\ y_1 \end{pmatrix} - \begin{pmatrix} x_2 \\ y_2 \end{pmatrix} = \begin{pmatrix} x_1 - x_2 \\ y_1 - y_2 \end{pmatrix} \]

If \( a = \begin{pmatrix} 4 \\ 5 \end{pmatrix} \), \( b = \begin{pmatrix} 2 \\ -1 \end{pmatrix} \)  and \( c = \begin{pmatrix} 0 \\ -1 \end{pmatrix} \)
(i) \[ a + b = \left( \frac{4}{5} + \frac{2}{1} \right) = \left( \frac{4+2}{5-1} \right) = \left( \frac{6}{4} \right) \]

(ii) \[ b - b = \left( \frac{2}{1} - \frac{4}{5} \right) = \left( \frac{2 - 4}{1 - 5} \right) = \left( \frac{-2}{-6} \right) \]

(iii) \[ \frac{1}{2} b - c = \frac{1}{2} \left( \frac{2}{-1} \right) - \left( \frac{0}{-1} \right) - \left( \frac{0}{-1} \right) = \left( \frac{2}{2} \right) - \left( \frac{-1}{2} \right) - \left( \frac{1}{2} \right) = \left( \frac{1}{0} \right) \]

(iv) \[ a - 2(b - c) = \left( \frac{4}{5} - \frac{2}{1} \right) - \left( \frac{0}{-1} \right) = \left( \frac{4}{5} - \frac{2}{1} \right) - \left( \frac{0}{-1} \right) = \left( \frac{6}{1} \right) \]

**Vector Geometry**

In the triangle ABC, \( \overrightarrow{AB} = a \) and \( \overrightarrow{BC} = b \). BD divides AC in the ratio 3:1. Find in terms of a and b:

(i) \( \overrightarrow{AC} \)

(ii) \( \overrightarrow{AD} \)

(iii) \( \overrightarrow{BD} \)

(i) \( \overrightarrow{AC} = \overrightarrow{AB} + \overrightarrow{BC} = a + b \)

(ii) Since BD divides AC in the ratio 3:1,

\[ \overrightarrow{AD} = \frac{3}{4} \overrightarrow{AC} = \frac{3}{4} (a + b) \]

(iii) \( \overrightarrow{BA} = -\overrightarrow{AB} = -a \), \( \overrightarrow{BD} = \overrightarrow{BA} + \overrightarrow{AD} = -a + \frac{3}{4} (a + b) \)

In quadrilateral OPQR, \( \overrightarrow{OP} = p \), \( \overrightarrow{OR} = r \), \( \overrightarrow{PQ} = p + r \), RS = 2OR. Find in terms of p and r:

(i) \( \overrightarrow{PR} \)

(ii) \( \overrightarrow{RQ} \)

(iii) \( \overrightarrow{RS} \)

(iv) \( \overrightarrow{QS} \)

(i) \( \overrightarrow{PR} = \overrightarrow{PO} + \overrightarrow{OR} = -p + r = r - p \)
(iii) $\overrightarrow{RS} = 2\overrightarrow{OR} = 2r$
(iv) $\overrightarrow{QS} = \overrightarrow{QR} + \overrightarrow{RS} = -2p + 2r = 2(r - p)$

In triangle OPQ, OP=p and OQ=q. R and S divide OP and OQ respectively in the ratio of 3:1.

(i) Find, in terms of p and q: $\overrightarrow{PQ}$, $\overrightarrow{OR}$, $\overrightarrow{OS}$ and $\overrightarrow{RS}$.

(ii) What can you deduce about RS and PQ?

Example: ABCD is a quadrilateral. P, Q, R and S are the mid-points of AB, BC, CD and DA respectively.

$\overrightarrow{AD} = a$ $\overrightarrow{AB} = b$ $\overrightarrow{BC} = c$

Find in terms of $a$, $b$ and $c$.

$\overrightarrow{AP}$, $\overrightarrow{CD}$, $\overrightarrow{CR}$, $\overrightarrow{PS}$, $\overrightarrow{QR}$, $\overrightarrow{PQ}$, $\overrightarrow{SR}$

What can you deduce about PQRS?

$\overrightarrow{AP} = \frac{1}{2} \overrightarrow{AB} = \frac{b}{2}$ $\overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AC} = b + c$

$\overrightarrow{CD} = \overrightarrow{CA} + \overrightarrow{AD} = -\overrightarrow{CA} + \overrightarrow{AD} = -(b + c) + a = a - b - c$

$\overrightarrow{CR} = \frac{1}{2} \overrightarrow{CD} = \frac{1}{2}(a - b - c)$ $\overrightarrow{PS} = \frac{1}{2}(\overrightarrow{PA} + \overrightarrow{AS}) = \frac{1}{2}(-b + a) = \frac{1}{2}(a - b)$
\[ \vec{QR} = \frac{1}{2}(\vec{QC} + \vec{CR}) = \frac{1}{2}(c + a - b - c) = \frac{1}{2}(a - b) \]

\[ \vec{PQ} = \frac{1}{2}(\vec{PB} + \vec{BQ}) = \frac{1}{2}(b + c) \quad \vec{SR} = \frac{1}{2}(\vec{SD} + \vec{DR}) = \frac{1}{2}[a - (a - b - c)] = \frac{1}{2}(b + c) \]

\[ \vec{PQRS} \text{ is a parallelogram.} \]

\[ \text{OPQRST is a regular hexagon.} \quad \vec{OP} = p \quad \vec{PQ} = q \quad \text{Find in terms of } p \text{ and } q \]

\[ \vec{RS}, \vec{ST}, \vec{QT}, \vec{QR}, \vec{PR}, \vec{OR} \]

\[ \text{Note: A regular hexagon has all sides of the same length, and all internal angles are 120 degrees. The longest diagonals of a regular hexagon, connecting diametrically opposite vertices, are twice the length of one side.} \]

\[ \vec{RS} = -\vec{OP} = -p \quad \vec{ST} = -\vec{PQ} = -q \quad \vec{QT} = -2p \quad \vec{QR} = q - p \]

\[ \vec{PR} = 2q - p \quad \vec{OR} = 2q \]
Narrated Anas (RA): The Prophet(SAW) used to say, “O Allah! Our Lord! Give us in this world that, which is good and in the Hereafter that, which is good and save us from the torment of the Fire” (Al-Bukhari)

Dear Brothers and Sisters!

I kindly request each one of you to remember me, my parents, my children, my family and the entire Muslims of the world in your daily Prayers. If you have not done it so far, then please do it now.

سلام عليكم و رحمت الله و بركاته

السلام عليكم و رحمت الله و بركاته

I kindly request each one of you to remember me, my parents, my children, my family and the entire Muslims of the world in your daily Prayers.

أبا عبدالله وردک

أبا عبدالله وردک

أبا عبدالله وردک

أبا عبدالله وردک

أبا عبدالله وردک

أبا عبدالله وردک

أبا عبدالله وردک

أبا عبدالله وردک

أبا عبدالله وردک

أبا عبدالله وردک

أبا عبدالله وردک

أبا عبدالله وردک

أبا عبدالله وردک

أبا عبدالله وردک

أبا عبدالله وردک

أبا عبدالله وردک

أبا عبدالله وردک

أبا عبدالله وردک

أبا عبدالله وردک

أبا عبدالله وردک

أبا عبدالله وردک

أبا عبدالله وردک

أبا عبدالله وردک

أبا عبدالله وردک

أبا عبدالله وردک

أبا عبدالله وردک

أبا عبدالله وردک

أبا عبدالله وردک

أبا عبدالله وردک

أبا عبدالله وردک

أبا عبدالله وردک

أبا عبدالله وردک

أبا عبدالله وردک

أبا عبدالله وردک

أبا عبدالله وردک

أبا عبدالله وردک

أبا عبدالله وردک

أبا عبدالله وردک

أبا عبدالله وردک

أبا عبدالله وردک

أبا عبدالله وردک

أبا عبدالله وردک

أبا عبدالله وردک

أبا عبدالله وردک

أبا عبدالله وردک

أبا عبدالله وردک

أبا عبدالله وردک

أبا عبدالله وردک

أبا عبدالله وردک

أبا عبدالله وردک

أبا عبدالله وردک

أبا عبدالله وردک

أبا عبدالله وردک

أبا عبدالله وردک

أبا عبدالله وردک

أبا عبدالله وردک

أبا Abdullahwardak53@gmail.com