Math Book GCSE
"In the name of Allah, the Most Beneficent, the Most Merciful."

Narrated Anas (RA): The Prophet (SAW) used to say, "O Allah! Our Lord! Give us in this world that, which is good and in the Hereafter that, which is good and save us from the torment of the Fire" (Al-Bukhari)

Dear Brothers and Sisters!

I kindly request each one of you to remember me, my parents, my children, my family and the entire Muslims of the world in your daily Prayers.

For me and for my family and for you and your family, may God bless us with His mercy.

Email: abdullahwardak53@gmail.com
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LESSON 1

RATIO

Example: In a gathering of Afghan tribal elders there was a mix of ages. The under 25s, the 25 - 40s and over 40s were in the ratio of 2: 3: 5

It was estimated that there were 2400 over 40s in the gathering. How many under 25s were there in the gathering?

Ans:
No of people per share = \( \frac{2400}{5} = 480 \)
Under 25s = \( 480 \times 2 = 960 \)
The 25 - 40s = \( 480 \times 3 = 1440 \)
Total = \( 960 + 1440 + 2400 = 4800 \)

Example: Three boys shared £48 in the ratio 5:4:3

Daniel received the smallest amount.
(a) Work out the amount Daniel received.

Ans: Daniel share = \( \frac{48}{(5 + 4 + 3)} \times 3 = £12 \)

A year ago, Daniel’s height was 1.24 meters. Daniel height has now increased by 9.5%.
(b) Work out Daniel’s height now.
Give your answer to an appropriate degree of accuracy.

Ans: Daniel height = \( 1.24 \times \left( \frac{109.5}{100} \right) = 1.3578 \ m \approx 1.36 \ m \)

Example: In a gathering of Afghan tribal elders there was a mix of educational background. The doctors, the engineers and the lawyers were in the ratio of 1.5: 2.5: 4.5

It was estimated that there were 300 doctors in the gathering. How many lawyers were there in the gathering?
Ans:
Lawyers = \frac{300}{1.5} \times 4.5 = \frac{1350}{1.5} = 900
Engineers = \frac{300}{1.5} \times 2.5 = \frac{750}{1.5} = 500
Doctors = \frac{300}{1.5} \times 1.5 = \frac{450}{1.5} = 300

Total = 900 + 500 + 300 = 1700

OR

Let the total number = x
Doctors = \frac{x}{1.5 + 2.5 + 4.5} \times 1.5 = 300
\frac{x}{8.5} \times 1.5 = 300
1.5x = 300 \times 8.5 = 2550
x = \frac{2550}{1.5} = 1700
Lawyers = \frac{1700}{8.5} \times 4.5 = 900
Engineers = \frac{1700}{8.5} \times 2.5 = 500
Doctors = \frac{1700}{8.5} \times 1.5 = 300

Example: In a gathering of Afghan tribal elders there doctors, engineers and lawyers. The doctors made up \frac{1}{5} of the gathering, the engineers made up \frac{2}{3} of the gathering. It was estimated that there were 300 lawyers in the gathering. How many doctors and engineers were there in the gathering?

Ans:
Lawyers made up \frac{1 - \left( \frac{1}{5} + \frac{2}{3} \right)}{15} = \frac{1 - \frac{3 + 10}{15}}{15} = \frac{13}{15} = \frac{15 - 13}{15} = \frac{2}{15}

Total number in the gathering = 300 \div \frac{2}{15} = 300 \times \frac{15}{2} = 2250
Engineers = $2250 \times \frac{2}{3} = 1500$

Doctors = $2250 \times \frac{1}{5} = 450$

Total = $300 + 1500 + 450 = 2250$

OR

Ratio of doctors, engineers, lawyers

Multiply each ratio by 15

Ratio of doctors, engineers, lawyers

Ratio of doctors, engineers, lawyers

Engineers = $\frac{300}{2} \times 10 = 1500$

Doctors = $\frac{300}{2} \times 3 = 450$

Total = $300 + 1500 + 450 = 2250$

OR

Let the total number = $x$

Lawyers = $\frac{x}{3+10+2} \times 2 = 300$

$\frac{x}{15} \times 2 = 300$

$2x = 15 \times 300 = 4500$

$x = \frac{4500}{2} = 2250$

Engineers = $\frac{2250}{15} \times 10 = 1500$

Doctors = $\frac{2250}{15} \times 3 = 450$

Lawyers = $\frac{2250}{15} \times 2 = 300$

Total = $300 + 1500 + 450 = 2250$

OR

Let the total number = $x$
Example: In a gathering of Afghan tribal elders there was a mix of educational background. The doctors, the engineers and the lawyers were in the ratio of 1.5: \( x + 1 : x \)

It was estimated that there were 500 engineers and 300 doctors in the gathering. How many people attended the gathering?

Ans:

Let the total number = \( y \)

Doctors = \( \frac{y}{1.5 + (x + 1) + x} \times 1.5 = 300 \) \( \cdots (1) \)

Engineers = \( \frac{y}{1.5 + (x + 1) + x} \times (x + 1) = 500 \) \( \cdots (2) \)

Doctors = \( \frac{1.5y}{2.5 + 2x} = 300 \) \( \cdots (1) \)

Engineers = \( \frac{y(x + 1)}{2.5 + 2x} = 500 \) \( \cdots (2) \)
Dividing Eq.1 by Eq.2:

\[
\frac{1.5y}{2.5 + 2x} \div \frac{y(x+1)}{2.5 + 2x} = \frac{300}{500}
\]

\[
\frac{1.5y}{2.5 + 2x} \times \frac{2.5 + 2x}{y(x+1)} = \frac{300}{500}
\]

\[
\frac{1.5y}{y(x+1)} = \frac{300}{500}
\]

\[
\frac{1.5}{x+1} = \frac{3}{5}
\]

\[
3(x+1) = 7.5
\]

\[
x + 3 = 7.5
\]

\[
x = 7.5 - 3 = 4.5
\]

\[
x = \frac{4.5}{3} = 1.5
\]

Doctors = \[
\frac{1.5y}{2.5 + 2x} = 300 \quad \text{...(1)}
\]

Doctors = \[
\frac{1.5y}{2.5 + 2 \times 1.5} = 300 \quad \text{...(1)}
\]

Doctors = \[
\frac{1.5y}{5.5} = 300 \quad \text{...(1)}
\]

Total = \[
y = \frac{5.5 \times 300}{1.5} = 1100
\]

Example: An inheritance is to be shared between three brothers, Hamid, Arif, and Bashir. They will stipulate that Arif should receive twice as much as Hamid but £8000 less than Bashir. The inheritance totals £52000.

(a) How much will each son receive?

Ans:

Let Bashir share = \[x\]

Then Arif share = \[(x - 800)\]

and Hamid share = \[\frac{1}{2}(x - 800)\]
\[ x + (x - 8000) + \frac{1}{2}(x - 8000) = 52000 \]
\[ 2x + 2(x - 8000) + (x - 8000) = 2 \times 52000 \]
\[ 2x + 2x - 16000 + x - 8000 = 104000 \]
\[ 5x = 104000 + 24000 \]
\[ 5x = 128000 \]
\[ x = \frac{128000}{5} = 25600 \]

**Bashir share**  \( x = 25600 \)

**Then Arif share**  \( (x - 800) = 25600 - 8000 = 17600 \)

**and Hamid share**  \( \frac{1}{2}(x - 800) = \frac{1}{2}(25600 - 8000) = 8800 \)

**OR**

Let **Hamid share**  \( = x \)

Then **Arif share**  \( = 2x \)

and **Bashir share**  \( = 2x + 8000 \)

\[ x + 2x + 2x + 8000 = 52000 \]
\[ 5x = 52000 - 8000 \]
\[ 5x = 44000 \]
\[ x = \frac{44000}{5} = 8800 \]

**Hamid share**  \( x = 8800 \)

**Arif share**  \( = 2x = 2 \times 8800 = 17600 \)

**and Bashir share**  \( = 2x + 8000 = 17600 + 8000 = 25600 \)

(b) In what ratio, in its simplest terms, is the money divided between Hamid, Arif, and Bashir respectively.

(Ans:  
LESSON 2

Making the Subject

Example: Make $a$ the subject of $r = 3a + 6$
Ans: 

\begin{align*}
  r &= 3a + 6 \\
  3a &= r - 6 \\
  a &= \frac{r - 6}{3}
\end{align*}

Example: Make $b$ the subject of $a = 3b^2 - 8$
Ans: 

\begin{align*}
  a &= 3b^2 - 8 \\
  3b^2 &= a + 8 \\
  b^2 &= \frac{a + 8}{3} \\
  b &= \pm \sqrt{\frac{a + 8}{3}}
\end{align*}

Example: Make $a$ the subject of $b = \pm \sqrt{\frac{a + 8}{3}}$
Ans: 

\begin{align*}
  b &= \pm \sqrt{\frac{a + 8}{3}} \\
  (b)^2 &= \left( \pm \sqrt{\frac{a + 8}{3}} \right)^2 \\
  b^2 &= \frac{a + 8}{3} \\
  3b^2 &= a + 8 \\
  a &= 3b^2 - 8
\end{align*}

Example: Make $y$ the subject of $x = \frac{21 - 5y}{2}$
Ans: 

\begin{align*}
  x &= \frac{21 - 5y}{2} \\
  2x &= 21 - 5y \\
  5y &= 21 - 2x \\
  y &= \frac{21 - 2x}{5}
\end{align*}

Example: Make $z$ the subject of $x = a\left( z - \frac{b}{c} \right)$
Ans: \[ x = a \left( \frac{b}{z - c} \right) \]
\[ x = az - \frac{ab}{c} \]
\[ az = x + \frac{ab}{c} \]
\[ z = \frac{x}{a} + \frac{ab}{ac} \]
\[ z = \frac{x}{a} + \frac{b}{c} \]

Example: Make \( r \) the subject of \( V = \frac{4}{3} \pi r^3 \)

Ans: \( V = \frac{4}{3} \pi r^3 \)
\[ r^3 = \frac{3V}{4\pi} \]
\[ \Rightarrow r = \sqrt[3]{\frac{3V}{4\pi}} \]

Example: Make \( r \) the subject of \( \sqrt[3]{V} = \frac{4}{3} \pi r^3 \)

Ans: \( \sqrt[3]{V} = \frac{4}{3} \pi r^3 \)
\[ \left( \sqrt[3]{V} \right)^2 = \left( \frac{4}{3} \pi r^3 \right)^2 \]
\[ \Rightarrow V = \frac{16}{9} \pi^2 r^6 \]
\[ r^6 = \frac{9V}{16\pi^2} \]
\[ \Rightarrow r = \sqrt[6]{\frac{9V}{16\pi^2}} \]

Example: Make \( z \) the subject of \( y - xy = 2x - 5 \)

Ans: \[ s = \frac{z + 1}{z - 1} \]
\[ sz - s = z + 1 \]
\[ sz - z = s + 1 \]
\[ z(s - 1) = s + 1 \]
\[ z = \frac{s + 1}{s - 1} \]
\[ z = \frac{s + 1}{s - 1} \]
Example: Make $T$ the subject of $I = V \left( 1 - e^{-\frac{RT}{L}} \right)$

Ans:

$$I = V \left( 1 - e^{-\frac{RT}{L}} \right)$$

$$I = VI - Ve^{-\frac{RT}{L}}$$

$$Ve^{-\frac{RT}{L}} = VI - I$$

$$e^{-\frac{RT}{L}} = I - \frac{I}{V} = \frac{VI - I}{V}$$

$$\ln \left( e^{-\frac{RT}{L}} \right) = \ln \left( \frac{VI - I}{V} \right)$$

$$-\frac{RT}{L} \ln(e) = \ln \left( \frac{VI - I}{V} \right)$$

$$-\frac{RT}{L} = \ln \left( \frac{VI - I}{V} \right)$$

$$T = -\frac{L}{R} \ln \left( \frac{VI - I}{V} \right)$$
LESSON 3

The Mean

To find the mean, you need to add up all the data, and then divide the total by the number of values in the data.

Example: Set A: 2, 2, 3, 5, 5, 7, 8.

Adding the numbers up gives: \[ 2 + 2 + 3 + 5 + 5 + 7 + 8 = 32 \]

There are 7 values, so you divide the total by 7: \[ \frac{32}{7} = 4.57142... \]

So the mean is 4.57 (2 d.p.)

\[ \text{Mean} = \frac{\sum x}{n} = \frac{2 + 2 + 3 + 5 + 5 + 7 + 8}{7} = \frac{32}{7} = 4.57142 = 4.57 \text{ (2} \text{d.p.)} \]

The Median

To find the median, you need to put the values in order, then find the middle value. If there are two values in the middle then you find the mean of these two values.

Example: The numbers in order: 2, 2, 3, 5, 5, 7, 8

The middle value is marked in red, and it is 5. So the median is 5

Example: The numbers in order: 2, 3, 3, 4, 6, 7

This time there are two values in the middle. They have been put in red. The median is found by calculating the mean of these two values: \( (3 + 4) ÷ 2 = 3.5 \). So the median is 3.5

The Mode

The mode is the value which appears the most often in the data. It is possible to have more than one mode if there is more than one value which appears the most.

Example: The data values: 2, 3, 3, 4, 6, 7.

There is only one value which appears most often - the number 3. It appears more times than any of the other data values. So the mode is 3

Example: The data values: 2, 2, 3, 5, 5, 7, 8.
The values which appear most often are 2 and 5. They both appear more time than any of the other data values. So the modes are 2 and 5.

**The Range**

To find the range, you first need to find the lowest and highest values in the data. The range is found by subtracting the lowest value from the highest value.

**Example:** The data values: \(2, 2, 3, 5, 5, 7, 8\)

The lowest value is 2 and the highest value is 8. Subtracting the lowest from the highest gives: \(8 - 2 = 6\). So the range is 6.

**Example:** The data values: \(2, 3, 3, 4, 6, 7\)

The lowest value is 2 and the highest value is 7. Subtracting the lowest from the highest gives: \(7 - 2 = 5\). So the range is 5.

**Example.** Use the list of numbers of the following numbers to find the mean, median and range.

\(-257\, 8,954\, 938\, -9,053\, -737\)

**Ans:**

\[
Mean = \frac{\sum x}{n} = \frac{-257 + 8954 + 938 - 9053 - 737}{5} = \frac{-155}{5} = -31
\]

For median, the data needs to be in either ascending or descending order first:

\(-9053\, -737\, -257\, 938\, 8954\)

\(-9053\, -737\, -257\, 938\, 8954\)

**Median** is the middle term: i.e. \(\left(\frac{n + 1}{2}\right)^{th}\) = \(\left(\frac{6 + 1}{2}\right)^{th}\) = \(\left(\frac{7}{2}\right)^{th}\) = 3\(^{rd}\) term = -257

Range = Max - Min = 8954 - (-9053) = 8954 + 9053 = 18007

**Example:** A school teacher has the following record about the Biology test results for his class.
(a) Which is the modal class group?
Ans:
*Modal class group is 41 - 50, because 9 students marked those score.*

(b) Calculate the estimated mean score.
Ans:
Take the midpoints of the marks into consideration. Also pay attention that some groups are different from the others like 61 - 80 and 81 - 100.

<table>
<thead>
<tr>
<th>Mark</th>
<th>Midpoint values</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>31 - 40</td>
<td>( \frac{40 + 31}{2} = \frac{71}{2} = 35.5 )</td>
<td>5</td>
</tr>
<tr>
<td>41 - 50</td>
<td>( \frac{50 + 41}{2} = \frac{91}{2} = 45.5 )</td>
<td>9</td>
</tr>
<tr>
<td>51 - 60</td>
<td>( \frac{60 + 51}{2} = \frac{111}{2} = 55.5 )</td>
<td>6</td>
</tr>
<tr>
<td>61 - 80</td>
<td>( \frac{80 + 61}{2} = \frac{141}{2} = 70.5 )</td>
<td>8</td>
</tr>
<tr>
<td>81 - 100</td>
<td>( \frac{100 + 81}{2} = \frac{181}{2} = 90.5 )</td>
<td>4</td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td>32</td>
</tr>
</tbody>
</table>

Estimated Mean = \( \frac{(35.5 \times 5) + (45.5 \times 6) + (55.5 \times 6) + (70.5 \times 8) + (90.5 \times 4)}{32} \)

Estimated Mean = \( \frac{1846}{32} = 57.6875 \approx 57.7 \)
Example: A school teacher has the following record about the Biology test results for his class.

<table>
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<td>6</td>
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<tr>
<td>61 - 80</td>
<td>8</td>
</tr>
<tr>
<td>81 - 100</td>
<td>4</td>
</tr>
<tr>
<td>Total</td>
<td>32</td>
</tr>
</tbody>
</table>

(c) Which is the modal class group?
Ans: Modal class group is 41 - 50, because 9 students marked those score.

(d) Calculate the estimated mean score.
Ans: Take the midpoints of the marks into consideration. Also pay attention that some groups are different from the others like 61 - 80 and 81 - 100.

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<td>(\frac{40 + 31}{2} = \frac{71}{2} = 35.5)</td>
<td>5</td>
</tr>
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<td>(\frac{50 + 41}{2} = \frac{91}{2} = 45.5)</td>
<td>9</td>
</tr>
<tr>
<td>51 - 60</td>
<td>(\frac{60 + 51}{2} = \frac{111}{2} = 55.5)</td>
<td>6</td>
</tr>
<tr>
<td>61 - 80</td>
<td>(\frac{80 + 61}{2} = \frac{141}{2} = 70.5)</td>
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</tr>
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<td>81 - 100</td>
<td>(\frac{100 + 81}{2} = \frac{181}{2} = 90.5)</td>
<td>4</td>
</tr>
<tr>
<td>Total</td>
<td>32</td>
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</tbody>
</table>

Estimated Mean = \(\frac{(35.5\times5) + (45.5\times6) + (55.5\times6) + (70.5\times8) + (90.5\times4)}{32}\)

Estimated Mean = \(\frac{1846}{32} = 57.6875 \approx 57.7\)
Example: The table shows information about the number of hours that 120 children watched television last week.

<table>
<thead>
<tr>
<th>Number of hours (h)</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 ≤ h ≤ 2</td>
<td>10</td>
</tr>
<tr>
<td>2 ≤ h ≤ 4</td>
<td>20</td>
</tr>
<tr>
<td>4 ≤ h ≤ 6</td>
<td>25</td>
</tr>
<tr>
<td>6 ≤ h ≤ 8</td>
<td>40</td>
</tr>
<tr>
<td>8 ≤ h ≤ 10</td>
<td>15</td>
</tr>
<tr>
<td>10 ≤ h ≤ 12</td>
<td>10</td>
</tr>
</tbody>
</table>

(a) Work out an estimate for the mean number of hours that the children watched television last week.

<table>
<thead>
<tr>
<th>Mark</th>
<th>Midpoint values (h)</th>
<th>Frequency (f)</th>
<th>f x h</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 ≤ h ≤ 2</td>
<td>( \frac{0+2}{2} = 1 )</td>
<td>10</td>
<td>10 x 1 = 10</td>
</tr>
<tr>
<td>2 ≤ h ≤ 4</td>
<td>( \frac{2+4}{2} = 3 )</td>
<td>20</td>
<td>20 x 3 = 60</td>
</tr>
<tr>
<td>4 ≤ h ≤ 6</td>
<td>( \frac{4+6}{2} = 5 )</td>
<td>25</td>
<td>25 x 5 = 125</td>
</tr>
<tr>
<td>6 ≤ h ≤ 8</td>
<td>( \frac{6+8}{2} = 7 )</td>
<td>40</td>
<td>40 x 7 = 280</td>
</tr>
<tr>
<td>8 ≤ h ≤ 10</td>
<td>( \frac{8+10}{2} = 9 )</td>
<td>15</td>
<td>15 x 9 = 135</td>
</tr>
<tr>
<td>10 ≤ h ≤ 12</td>
<td>( \frac{10+12}{2} = 11 )</td>
<td>10</td>
<td>10 x 11 = 110</td>
</tr>
<tr>
<td>Total</td>
<td>( \sum f = 120 )</td>
<td>( \sum f h = 720 )</td>
<td></td>
</tr>
</tbody>
</table>

Estimated mean = \( \frac{\sum f \times h}{\sum f} = \frac{720}{120} = 6 \text{hrs} \)

(b) Complete the cumulative frequency table.
<table>
<thead>
<tr>
<th>Number of hours (h)</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>$0 \leq h &lt; 2$</td>
<td>10</td>
</tr>
<tr>
<td>$0 \leq h &lt; 4$</td>
<td></td>
</tr>
<tr>
<td>$0 \leq h &lt; 6$</td>
<td></td>
</tr>
<tr>
<td>$0 \leq h &lt; 8$</td>
<td></td>
</tr>
<tr>
<td>$0 \leq h &lt; 10$</td>
<td></td>
</tr>
<tr>
<td>$0 \leq h &lt; 12$</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Mark</th>
<th>Frequency</th>
<th>Commulative frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>$0 \leq h &lt; 2$</td>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>$2 \leq h &lt; 4$</td>
<td>20</td>
<td>10 + 20 = 30</td>
</tr>
<tr>
<td>$4 \leq h &lt; 6$</td>
<td>25</td>
<td>30 + 25 = 55</td>
</tr>
<tr>
<td>$6 \leq h &lt; 8$</td>
<td>40</td>
<td>55 + 40 = 95</td>
</tr>
<tr>
<td>$8 \leq h &lt; 10$</td>
<td>15</td>
<td>95 + 15 = 110</td>
</tr>
<tr>
<td>$10 \leq h &lt; 12$</td>
<td>10</td>
<td>110 + 10 = 120</td>
</tr>
</tbody>
</table>

**Note:** the ends of each interval as the length of journey (miles). For example, $0 \leq h < 2$, take 2, $2 \leq h < 4$, take 4, $4 \leq h < 6$, take 6, and so on.
(c) On the grid, draw a cumulative frequency graph for your table.

(d) Use your graph to find an estimate for the number of children who watched television for fewer than 5 hours last week.

**Example:** The table shows information about the number of hours that 40 children watched television last week.

<table>
<thead>
<tr>
<th>Number of hours (h)</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>$0 \leq h &lt; 1$</td>
<td>3</td>
</tr>
<tr>
<td>$1 \leq h &lt; 2$</td>
<td>8</td>
</tr>
<tr>
<td>$2 \leq h &lt; 3$</td>
<td>7</td>
</tr>
<tr>
<td>$3 \leq h &lt; 4$</td>
<td>10</td>
</tr>
<tr>
<td>$4 \leq h &lt; 5$</td>
<td>12</td>
</tr>
<tr>
<td>Total</td>
<td>40</td>
</tr>
</tbody>
</table>
Ans: Median is \( \frac{1}{2} (n + 1)^{th} \) term = \( \frac{1}{2} (40 + 1)^{th} \) term = (20.5)\(^{th}\) term

\( \Rightarrow \) It is between 20\(^{th}\) term and 21\(^{th}\) term \( \Rightarrow 3 \leq h \pi 4 \) is the class interval.

(b) Work out an estimate for the mean number of hours that the children watched television last week.

<table>
<thead>
<tr>
<th>Number of hours (h)</th>
<th>Mid-Point Value (h)</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 ≤ h ≤ 1</td>
<td>( \frac{0+1}{2} = 0.5 )</td>
<td>3</td>
</tr>
<tr>
<td>1 ≤ h ≤ 2</td>
<td>( \frac{1+2}{2} = 1.5 )</td>
<td>8</td>
</tr>
<tr>
<td>2 ≤ h ≤ 3</td>
<td>( \frac{2+3}{2} = 2.5 )</td>
<td>7</td>
</tr>
<tr>
<td>3 ≤ h ≤ 4</td>
<td>( \frac{3+4}{2} = 3.5 )</td>
<td>10</td>
</tr>
<tr>
<td>4 ≤ h ≤ 5</td>
<td>( \frac{4+5}{2} = 4.5 )</td>
<td>12</td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td>40</td>
</tr>
</tbody>
</table>

Estimated Mean = \( \frac{3 \times 0.5 + (8 \times 1.5) + (7 \times 2.5) + (10 \times 3.5) + (12 \times 4.5)}{40} \)

Estimated Mean = \( \frac{1.5 + 12 + 17.5 + 35 + 54}{40} = \frac{120}{40} = 3 \)
LESSON 4

Stem and Leaf Diagram:

A stem and leaf diagram is a diagram that shows data in a systematic way. The mode, median and range of data can be found easily from a stem and leaf diagram. Consider the following examples.

Example. Here are the speeds, in miles per hour, of 16 cars.

<p>| | | | | | | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>31</td>
<td>52</td>
<td>43</td>
<td>49</td>
<td>36</td>
<td>35</td>
<td>33</td>
<td>29</td>
<td>54</td>
<td>43</td>
</tr>
<tr>
<td>43</td>
<td>44</td>
<td>46</td>
<td>42</td>
<td>39</td>
<td>55</td>
<td>48</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Draw an ordered stem and leaf diagram for these speeds.

Ans:

<table>
<thead>
<tr>
<th>Stem</th>
<th>Leaves</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>9</td>
</tr>
<tr>
<td>3</td>
<td>1 3 5 6 9</td>
</tr>
<tr>
<td>4</td>
<td>2 3 3 4 6 8 9</td>
</tr>
<tr>
<td>5</td>
<td>2 4 5</td>
</tr>
</tbody>
</table>

Key: 2|9 Means 29 mph

Note: In this case, the key makes it clear that the stem shows the tens and the leaves show the units. The data is written so that every number has a tens and units value.

Example. Here are the speeds, in miles per hour, of 16 cars.

<p>| | | | | | | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>3.1</td>
<td>5.2</td>
<td>4.3</td>
<td>4.9</td>
<td>3.6</td>
<td>3.5</td>
<td>3.3</td>
<td>2.9</td>
<td>5.4</td>
<td>4.3</td>
</tr>
<tr>
<td>4.4</td>
<td>4.6</td>
<td>4.2</td>
<td>3.9</td>
<td>5.5</td>
<td>4.8</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Draw an ordered stem and leaf diagram for these speeds.

Ans:
Note: In this case, the key makes it clear that the stem shows the tens and the leaves show the units. The data is written so that every number has a tens and units value.

Example. Complete a stem-and-leaf plot for the following list of times in seconds:
7.6, 8.1, 9.2, 6.8, 5.9, 6.2, 6.1, 5.8, 7.3, 8.1, 8.8, 7.4, 7.7, 8.2
Ans:
First, I'll reorder this list:
5.8, 5.9, 6.1, 6.2, 6.8, 7.3, 7.4, 7.6, 7.7, 8.1, 8.1, 8.2, 8.8, 9.2

These values have one decimal place, but the stem-and-leaf plot makes no accommodation for this. The stem-and-leaf plot only looks at the last digit (for the leaves) and all the digits before (for the stem). So I'll have to put a "key" or legend on this plot to show what I mean by the numbers in this plot. The ones digits will be the stem values, and the tenths will be the leaves.

Example: Complete a stem-and-leaf plot for the following list of values:
If I try to use the last digit, the hundredths digit, for these numbers, the stem-and-leaf plot will be enormously long, because these values are so spread out. (With the numbers' first three digits ranging from 232 to 270, I'd have thirty-nine leaves, most of which would be empty.) So instead of working with the given numbers, I'll round each of the numbers to the nearest tenth, and then use those new values for my plot. Rounding gives me the following list:

23.3, 24.1, 24.8, 24.8, 25.0, 25.3, 25.6, 25.9, 26.3, 26.3, 27.1

Then my plot looks like this:

<table>
<thead>
<tr>
<th>Stem</th>
<th>Leaves</th>
</tr>
</thead>
<tbody>
<tr>
<td>23</td>
<td>3</td>
</tr>
<tr>
<td>24</td>
<td>1 8 8</td>
</tr>
<tr>
<td>25</td>
<td>0 3 6 9</td>
</tr>
<tr>
<td>26</td>
<td>3 3</td>
</tr>
<tr>
<td>27</td>
<td>1</td>
</tr>
</tbody>
</table>

Key 23 3 Means 23.3 Seconds

Example.  A teacher recorded the scores in a Mental Maths test for her year 10 class.
19, 46, 27, 35, 11, 13, 22, 34, 18, 26, 32, 44, 34, 16
Construct a stem and leaf diagram to represent this information.

Ans:

<table>
<thead>
<tr>
<th>Stem</th>
<th>Leaves</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1 3 6 8 9</td>
</tr>
<tr>
<td>2</td>
<td>2 6 7</td>
</tr>
<tr>
<td>3</td>
<td>2 4 4 5</td>
</tr>
<tr>
<td>4</td>
<td>4 6</td>
</tr>
</tbody>
</table>

Key 3 2 Means 32 Marks

Example.  A teacher recorded the scores in a Mental Maths test for her year 10 class.
19.6, 46.3, 27.7, 35, 11, 13, 22, 34, 18, 26, 32, 44, 34, 16??
Construct a stem and leaf diagram to represent this information.
Ans:

<table>
<thead>
<tr>
<th>Stem</th>
<th>Leaves</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>1 3 6 8 9</td>
</tr>
<tr>
<td>20</td>
<td>2 6 7</td>
</tr>
<tr>
<td>30</td>
<td>2 4 4 5</td>
</tr>
<tr>
<td>40</td>
<td>4 6</td>
</tr>
</tbody>
</table>

Key 3 2 Means 32 Marks

Example:

<table>
<thead>
<tr>
<th>Man</th>
<th>Women</th>
</tr>
</thead>
<tbody>
<tr>
<td>21</td>
<td>28 30 31 32 32 33 33 34 34 35 35 35 36 36 41 41 42 42 45 46 51</td>
</tr>
<tr>
<td>12</td>
<td>14 14 15 15 20 21 23 23 28 32 32 33 33 34 34 35 35 35 35 36 36 38 38 40 43 44</td>
</tr>
</tbody>
</table>

Back-to-back stem and leaf diagrams can be used to compare 2 sets of data relating to the same subject. In our example we could add the data for 30 women

<table>
<thead>
<tr>
<th>Women</th>
<th>Leaves</th>
<th>Stem</th>
<th>Leaves</th>
<th>Men</th>
</tr>
</thead>
<tbody>
<tr>
<td>4 4 2</td>
<td>1</td>
<td>5 5 1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3 3 1 0</td>
<td>2 1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>8 2 8</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4 4 4 3 3 3 2 2 2</td>
<td>3 0 0 1 1 2 2 2 2 3 3 3 4 4 4</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>8 8 6 6 5 5 5 5 5</td>
<td>3 5 5 5 5 6 6</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0 4 1 1 2</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4 3 4 5 6</td>
<td>Key 3 5 Means 35 Marks</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Key 5 3 Means 35 Marks
Case-1: \( n = \text{odd} \)

<table>
<thead>
<tr>
<th>Lower 7 marks</th>
<th>Upper 7 marks</th>
</tr>
</thead>
<tbody>
<tr>
<td>10 39 40 43 45 47 48 49</td>
<td>50 50 51 52 56 58 82</td>
</tr>
</tbody>
</table>

There are 15 items of data, so the median is the 8th item, which is 49.

The upper quartile is the median of the upper 7 marks, which is 52.

---

Case-2: \( n = \text{even}, \text{and} \ n/2 = \text{even} \)

<table>
<thead>
<tr>
<th>Lower 8 marks</th>
<th>Upper 8 marks</th>
</tr>
</thead>
<tbody>
<tr>
<td>10 39 40 43 45 47 48 49</td>
<td>50 50 51 52 56 58 82</td>
</tr>
</tbody>
</table>

There are 16 items of data, so the median is the 8th item, which is 49.

The upper quartile is the median of the upper 8 marks, which is 54.

---

Case-3: \( n = \text{even}, \text{and} \ n/2 = \text{odd} \)

<table>
<thead>
<tr>
<th>Lower 11 terms</th>
<th>Upper 11 terms</th>
</tr>
</thead>
<tbody>
<tr>
<td>10 39 40 43 45 47 48 49 50 51 52 56 58 59 60 62 70 73 85 87</td>
<td></td>
</tr>
</tbody>
</table>

There are 22 items of data, so the median is the 11th item, which is 51.

The upper quartile is the median of the upper 11 terms, which is 62.

---

Example: A stem and leaf diagram of some data is shown below.
Stem | Leaves
--- | ---
10 | 0 1
15 | 1 2 2 3 3 3
20 | 0 2 3 4 4
25 | 0 1 4
30 | 3
35 | 2 3
40 | 0

(i) Describe shape of the distribution

(ii) Find the values of

(a) the median,

\[ \text{Ans:} \quad \text{The median is the } \left(\frac{n+1}{2}\right) \text{ piece of data. (If necessary, average 2 data items).} \]

\[ \Rightarrow \left(\frac{n+1}{2}\right)^{th} = \left(\frac{20+1}{2}\right)^{th} = \left(\frac{21}{2}\right)^{th} = (10.5)^{th} \text{ term.} \]

\[ \Rightarrow \text{Median} = \frac{10^{th} \text{ term} + 11^{th} \text{ term}}{2} = \frac{202 + 203}{2} = \frac{405}{2} = 202.5 \]

(b) the range \quad \text{Ans:} \quad \text{Range} = 400 - 100 = 300

(c) the midrange \quad \text{Ans:} \quad \text{Mid – Range} = \frac{100 + 400}{2} = \frac{500}{2} = 250

(d) the mode \quad \text{Ans:} \quad \text{Mode} = 153

(e) the interquartile range.

\[ \text{Ans:} \quad \text{The quartiles are found at the } \left(\frac{n+1}{4}\right)^{th} \text{ position and the } \left(\frac{3(n+1)}{4}\right)^{th} \text{ position.} \]

The lower quartile is at the \( \left(\frac{n+1}{4}\right)^{th} \) position:

\[ \Rightarrow \left(\frac{n+1}{4}\right)^{th} \text{ term} = \left(\frac{20+1}{4}\right)^{th} \text{ term} = \left(\frac{21}{4}\right)^{th} \text{ term} = 5.25^{th} \text{ term}. \]
\[ \Rightarrow \text{Lower quartile } LQ = \left( \frac{5^{th} \text{ term} + 6^{th} \text{ term}}{2} \right) = \left( \frac{152 + 153}{2} \right) = 152.5 \]

The upper quartile is at the \( \left( \frac{3(n + 1)}{4} \right) \) position:

\[ \Rightarrow \left( \frac{3(n + 1)}{4} \right)^{th} \text{ term} = \left( \frac{3(20 + 1)}{4} \right)^{th} \text{ term} = \left( \frac{63}{4} \right)^{th} \text{ term} = 15.75^{th} \text{ term}. \]

\[ \Rightarrow \text{Upper quartile } UQ = \left( \frac{15^{th} \text{ term} + 16^{th} \text{ term}}{2} \right) = \left( \frac{251 + 254}{2} \right) = 252.5 \]

Example. A stem and leaf diagram of some data is shown below.

<table>
<thead>
<tr>
<th>Stem</th>
<th>Leaves</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>0 1</td>
</tr>
<tr>
<td>15</td>
<td>1 2 2 3 3 3</td>
</tr>
<tr>
<td>20</td>
<td>0 2 3 4 4</td>
</tr>
<tr>
<td>25</td>
<td>0 1 4</td>
</tr>
<tr>
<td>30</td>
<td>3</td>
</tr>
<tr>
<td>35</td>
<td>2 3</td>
</tr>
<tr>
<td>40</td>
<td>0 3</td>
</tr>
</tbody>
</table>

Key: 25\|1 represents 251

(ii) Describe shape of the distribution

(ii) Find the values of

(a) the median,
Ans: The median is the \( \left( \frac{n+1}{2} \right)^{th} \) term of data. (If necessary, average 2 data items). \( \Rightarrow \left( \frac{n+1}{2} \right)^{th} = \left( \frac{21+1}{2} \right)^{th} = \left( \frac{22}{2} \right)^{th} = 11^{th} \) term.

\( \Rightarrow \text{Median} = 11^{th} \text{term} = 203 \)

(b) the midrange , Ans: \( \text{Mid – Range} = \frac{100 + 403}{2} = \frac{503}{2} = 251.5 \)

(c) the mode, Ans: \( \text{Mode} = 153 \)

(d) the interquartile range.

Ans: The quartiles are found at the \( \left( \frac{n+1}{4} \right)^{th} \) term and the \( \left( \frac{3(n+1)}{4} \right)^{th} \) term.

The lower quartile is at the \( \left( \frac{n+1}{4} \right)^{th} \) term:

\( \Rightarrow \left( \frac{n+1}{4} \right)^{th} \text{term} = \left( \frac{21+1}{4} \right)^{th} \text{term} = \left( \frac{22}{4} \right)^{th} \text{term} = 5.5^{th} \text{term} . \)

\( \Rightarrow \text{Lower quartile} = LQ = \left( \frac{5^{th} \text{term} + 6^{th} \text{term}}{2} \right) = \left( \frac{152 + 153}{2} \right) = 152.5 \)

The upper quartile is at the \( \left( \frac{3(n+1)}{4} \right)^{th} \) term:

\( \Rightarrow \left( \frac{3(n+1)}{4} \right)^{th} \text{term} = \left( \frac{3(21+1)}{4} \right)^{th} \text{term} = \left( \frac{66}{4} \right)^{th} \text{term} = 16.5^{th} \text{term} . \)

\( \Rightarrow \text{Upper quartile} = UQ = \left( \frac{16^{th} \text{term} + 17^{th} \text{term}}{2} \right) = \left( \frac{254 + 303}{2} \right) = 278.5 \)

Box-and-whisker plots
Example: A camera records the speeds in miles per hour of 15 vehicles on a motorway. The speeds are given below.

73  67  75  64  52  63  75  81  77  72  68  74  79  72  71

(i) Construct a stem and leaf diagram to represent these data.

Ans:

<table>
<thead>
<tr>
<th>Stem</th>
<th>Leaves</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>2</td>
</tr>
<tr>
<td>6</td>
<td>3  4  7</td>
</tr>
<tr>
<td>7</td>
<td>1  2  2</td>
</tr>
<tr>
<td>8</td>
<td>1</td>
</tr>
</tbody>
</table>

Key 6 3 represents 63

(ii) Write down the median, range and midrange of the data.

Ans: The median is the \( \left( \frac{n+1}{2} \right) \) th term = \( \left( \frac{21+1}{2} \right) \) th term = 11 th term.

\[ \Rightarrow \text{Median} = 11 \text{th term} = 72. \]

Range = 81 – 52 = 29

Mid – Range = \( \frac{52 + 81}{2} \) = \( \frac{133}{2} \) = 66.5

(ii) Which of the median and midrange would you recommend to measure the central tendency of the data? Briefly explain your answer.

Example. (a) Find the interquartile range of the set of marks below from a test taken by 15 students.

50  82  40  51  45  50  48  49  47  10  43  58  56  52  39

(b) One student was absent and took the test the following week, scoring 59. Find the new interquartile range.

Ans:
(a) First arrange the data in order of size:

```
10  39  40  43  45  47  48  49  50  50  51  52  56  58  82
```

Lower 7 marks | Upper 7 marks
---|---
10  39  40  43  45  47  48  49  50  50  51  52  56  58  82 | 10  39  40  43  45  47  48  49  50  50  51  52  56  58  82

The lower quartile is the median of the lower 7 marks, which is 43.
There are 15 items of data, so the median is the 8th item, which is 49.
The upper quartile is the median of the upper 7 marks, which is 52.

(b) The new set of data has 16 items:

```
10  39  40  43  45  47  48  49  50  50  51  52  56  58  59  82
```

The lower quartile is the median of the lower 8 marks, which is \((43+45)/2 = 44\).
For an even number of data items, the median falls between two items of data, so the median is \((49+50)/2 = 49.5\).
The upper quartile is the median of the upper 8 marks, which is \((52+56)/2 = 54\).

Look at this:

```
10  39  40  43  45  47  48  49  50  50  51  52  56  58  59  82
```
The lower quartile is the median of the lower 8 marks, which is \((43+45)/2=44\).

There are 16 items of data, so the median is \((49+50)/2 = 49.5\).

The upper quartile is the median of the upper 8 marks, which is \((52+56)/2 = 54\).

Consider the following Box-and-Whisker Plot

Example: Estimated speeds of 30 vehicles travelling along a stretch of road are shown below.
Stem | Leaves
---|---
1 | 4
2 | 4 6 6 8 8
3 | 0 2 4 4 6 8 8 8
4 | 0 0 2 2 2 4 8
5 | 0 2 6 6 8
6 | 4
7 | 0 4
8 | 2

Key 6 | 4 Means 64 mph

(a) Describe shape of the distribution
(b) Find the median and quartiles and draw a box plot.
(c) Work out the mean. (Given \( \sum x = 1268 \)).

Ans: \( n = 30 \) = even and \( \frac{n}{2} = \frac{30}{2} = 15 \) = odd

(a) Positive skew? Find why?

(b) The median is the \( \left( \frac{n+1}{2} \right)^{th} \) term = \( \left( \frac{30+1}{2} \right)^{th} \) term = 15.5\(^{th}\) term.

\[ \Rightarrow \text{Median} = \left( \frac{15^{th} \text{ term} + 16^{th} \text{ term}}{2} \right) = \left( \frac{40 + 40}{2} \right) = 40 \]

The lower quartile is the median of the lower 15 terms

\[ \Rightarrow LQ = \frac{1}{2} (15 + 1)^{th} \text{ term} = 8^{th} \text{ term} = 32 \]

The upper quartile is the median of the upper 15 terms

\[ \Rightarrow UQ = \left[ 15 + \frac{1}{2} (15 + 1) \right]^{th} \text{ term} = 23^{rd} \text{ term} = 52 \]

Example:
### Example:
The box plot gives information about the distribution of the weights of bags on a plane.

(a) Jean says the heaviest bag weighs 23 kg.
She is **wrong**.
Explain why.
**Ans:** Heaviest bag is **29kg**

(b) Write down the median weight.
**Ans:** The median is **17**

(c) Work out the interquartile range of the weights.
**Ans:**
The lower interquartile is 10
The upper interquartile is 23
The interquartile range is 23 - 10 = 13
There are 240 bags on the plane.
(d) Work out the number of bags with a weight of 10 kg or less.

Ans:
$LQ = 10 \Rightarrow LQ = \frac{1}{4}(240) = 60$th bag $\Rightarrow$ there are 60 bags less than 10 kg

Example: Harry grows tomatoes and weighed 60 tomatoes. The cumulative frequency graph shows some information about these weights.

(a) Use the graph to find an estimate for the median weight. The 60 tomatoes had a minimum weight of 153 grams and a maximum weight of 186 grams.

(b) Use this information and the cumulative frequency graph to draw a box plot for the 60 tomatoes.

Example: The time taken by each of 100 people to complete a task is shown cumulatively in the table.
(a) Draw a cumulative graph on the grid below to illustrate this data.

\[\text{Cumulative frequency}\]

\[
\begin{array}{|c|c|c|c|c|c|c|c|c|}
\hline
\text{Time (mins)} & \leq 1 & \leq 2 & \leq 3 & \leq 4 & \leq 5 & \leq 6 & \leq 7 & \leq 8 & \leq 9 & \leq 10 \\
\hline
\text{Number} & 11 & 14 & 21 & 30 & 42 & 55 & 69 & 81 & 92 & 100 \\
\hline
\end{array}
\]

(b) What is the median time?

\[\text{Ans:}\]

The median position is given by \(\frac{1}{2}(n + 1) = \frac{1}{2}(100 + 1) = \frac{101}{2} = 50.5\), so from the graph:

The median time is: **5.7 minutes**.

(c) What is the interquartile range?

The lower quartile position is given by:

\[\frac{1}{4}(n + 1) = \frac{1}{4}(100 + 1) = \frac{101}{4} = 25.25^{th}\text{ value} = 3.3 \text{ minutes}\]

The upper quartile position is given by:

\[\frac{3}{4}(n + 1) = \frac{3}{4}(100 + 1) = \frac{303}{4} = 75.75^{th}\text{ value} = 7.4 \text{ minutes}\]

The interquartile range is: **7.4 - 3.3 = 4.1 minutes**.
(d) The quickest 35 people went on to do a further test. What was the longest time that one of those 35 people took to complete the first task?  
**Ans:** Read from 35 on the graph to get 4.4 minutes

**Example:** In a survey, 200 people under the age of 25 were asked how many hours of sleep they had during the past week. The results of the survey are shown in the table below.

<table>
<thead>
<tr>
<th>Hours of sleep (h)</th>
<th>Frequency</th>
<th>Cumulative Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>h≤35</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>35&lt;h≤40</td>
<td>7</td>
<td>7</td>
</tr>
<tr>
<td>40&lt;h≤45</td>
<td>10</td>
<td>17</td>
</tr>
<tr>
<td>45&lt;h≤50</td>
<td>16</td>
<td>33</td>
</tr>
<tr>
<td>50&lt;h≤55</td>
<td>69</td>
<td>102</td>
</tr>
<tr>
<td>55&lt;h≤60</td>
<td>64</td>
<td>166</td>
</tr>
<tr>
<td>60&lt;h≤65</td>
<td>29</td>
<td>195</td>
</tr>
<tr>
<td>65&lt;h≤70</td>
<td>5</td>
<td>200</td>
</tr>
</tbody>
</table>

**Example:** Here is some information about the time, in minutes, it took the 21 teachers at a school to get to work on Monday.

13 18 20 35 45 34 44 23 33 12 46 21 22 17 22 31 23 8 15 22 10

(a) Draw an ordered stem and leaf diagram to show this information.
Road works near the school meant that the time to travel to school by every teacher on Tuesday was increased by 5 minutes.

(b) What was the median of the times on Tuesday?

Ans: For finding median, use this:

$$\frac{n+1}{2} = \frac{21+1}{2} = \frac{22}{2} = 11^{th} \text{ term (i.e. old median = 22)}$$

New Median = $22 + 5 = 27$

(c) State whether the interquartile range of the times on Tuesday would be less, greater or the same as the interquartile range of the times on Monday. Give a reason for your answer.

Example: A GCSE geography student is investigating a claim that global warming is causing summers in Britain to have more rainfall. He collects rainfall data from a local weather station for 2001 and 2006. The vertical line chart shows the number of days per week on which some rainfall was recorded during the 22 weeks of summer 2001.

(i) Show that the median of the data is 4, and find the interquartile range.
<table>
<thead>
<tr>
<th>Number of days per week with rain</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>5</td>
<td>3</td>
</tr>
<tr>
<td>6</td>
<td>2</td>
</tr>
<tr>
<td>7</td>
<td>2</td>
</tr>
</tbody>
</table>

\[ \sum f = 22 \]

For finding median, use this:

\[ \frac{n + 1}{2} = \frac{22 + 1}{2} = 11.5^{th} \text{ term} \]

(\[ i.e. \text{ median} = Q_2 = \frac{11^{th} \text{ term} + 12^{th} \text{ term}}{2} = \frac{4 + 4}{2} = 4 \])

The lower quartile is the median of the lower 11 days:

\[ \frac{n + 1}{2} = \frac{11 + 1}{2} = 6^{th} \text{ term} \]

(\[ i.e. \text{ lower quartile} = LQ = Q_1 = 6^{th} \text{ term} = 2 \])

The upper quartile is the median of the upper 11 days:

\[ 11 + 6 = 17^{th} \text{ term} \]

(\[ i.e. \text{ upper quartile} = UQ = Q_3 = 17^{th} \text{ term} = 5 \])

The interquartile range = \[ Q_3 - Q_1 = 5 - 2 = 3 \]

**Example:**

A taxi driver operates from a taxi rank at a main railway station in London. During one particular week he makes 120 journeys, the lengths of which are summarised in the table.

<table>
<thead>
<tr>
<th>Length (x miles)</th>
<th>[ 0 \pi x \leq 1 ]</th>
<th>[ 1 \pi x \leq 2 ]</th>
<th>[ 2 \pi x \leq 3 ]</th>
<th>[ 3 \pi x \leq 4 ]</th>
<th>[ 4 \pi x \leq 6 ]</th>
<th>[ 6 \pi x \leq 10 ]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of journeys</td>
<td>38</td>
<td>30</td>
<td>21</td>
<td>14</td>
<td>9</td>
<td>8</td>
</tr>
</tbody>
</table>

(i) Draw a cumulative frequency diagram to illustrate the data.

(ii) Use your graph to estimate the median length of journey and the quartiles. Hence find the interquartile range.

(iii) State the type of skewness of the distribution of the data.
Example: The times taken, in minutes, by 80 people to complete a crossword puzzle are summarised by the box and whisker plot below.

(i) Write down the range and the interquartile range of the times.
Ans: The range = 55 - 15 = 40
      The interquartile range = 35 - 26 = 9

(ii) Describe the shape of the distribution of the times.
Ans: Positively skewed. Middle 50% of data is closely bunched.
Example: At East Cornwall College, the mean GCSE score of each student is calculated. This is done by allocating a number of points to each GCSE grade in the following way.

<table>
<thead>
<tr>
<th>Grade</th>
<th>A*</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
<th>G</th>
<th>U</th>
</tr>
</thead>
<tbody>
<tr>
<td>Points</td>
<td>8</td>
<td>7</td>
<td>6</td>
<td>5</td>
<td>4</td>
<td>3</td>
<td>2</td>
<td>1</td>
<td>0</td>
</tr>
</tbody>
</table>

(i) Calculate the mean GCSE score, \( X \), of a student who has the following GCSE grades: A*, A*, A, A, A, B, B, B, B, C, D.

\[
\text{Ans: } \quad \text{Mean score} = \frac{2 \times 8 + 3 \times 7 + 4 \times 6 + 5 + 4}{11} = 6.36
\]

Example: 60 students study AS Mathematics at the college. The mean GCSE scores of these students are summarised in the table below.

<table>
<thead>
<tr>
<th>Mean GCSE score</th>
<th>Number of students</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.5 ( \leq x ) 5.5</td>
<td>8</td>
</tr>
<tr>
<td>5.5 ( \leq x ) 6.0</td>
<td>14</td>
</tr>
<tr>
<td>6.0 ( \leq x ) 6.5</td>
<td>19</td>
</tr>
<tr>
<td>6.5 ( \leq x ) 7.0</td>
<td>13</td>
</tr>
<tr>
<td>7.0 ( \leq x ) 8.0</td>
<td>6</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Mean GCSE score</th>
<th>Number of students</th>
<th>Class width</th>
<th>Frequency Density</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.5 ( \leq x ) 5.5</td>
<td>8</td>
<td>1</td>
<td>8 + 1 = 8</td>
</tr>
<tr>
<td>5.5 ( \leq x ) 6.0</td>
<td>14</td>
<td>0.5</td>
<td>14 ( \times ) 0.5 = 28</td>
</tr>
<tr>
<td>6.0 ( \leq x ) 6.5</td>
<td>19</td>
<td>0.5</td>
<td>19 ( \times ) 0.5 = 38</td>
</tr>
<tr>
<td>6.5 ( \leq x ) 7.0</td>
<td>13</td>
<td>0.5</td>
<td>13 ( \times ) 0.5 = 26</td>
</tr>
<tr>
<td>7.0 ( \leq x ) 8.0</td>
<td>6</td>
<td>1</td>
<td>6 + 1 = 6</td>
</tr>
</tbody>
</table>

(ii) Draw a histogram to illustrate this information.

Example: The total annual emissions of carbon dioxide, \( x \) tonnes per person, for 13 European countries are given below.

6.2  6.7  6.8  8.1  8.1  8.5  8.6  9.0  9.9  10.1  11.0  11.8  22.8

(i) Find the mean, median and midrange of these data.

\[
\text{Ans: } \quad \text{Mean} = \frac{6.2 + 6.7 + 6.8 + 8.1 + 8.1 + 8.5 + 8.6 + 9.0 + 9.9 + 10.1 + 11.0 + 11.8 + 22.8}{13} = \frac{127.6}{13} = 9.82
\]

Note: Because the data is already in rank order, then it does not need rearranging.
The **median** is the mid-value when the data are presented in rank order; it is the value of the $\frac{n+1}{2}$th item of n data items.

$$median = \frac{n+1}{2} \text{th term} = \frac{13+1}{2} = 7\text{th term of the data} = 8.6$$

The mid-range $= \frac{\text{Min} + \text{Max}}{2} = \frac{6.2 + 22.8}{2} = 14.5$

**Example:** The numbers of absentees per day from Mrs Smith’s reception class over a period of 50 days are summarised below.

<table>
<thead>
<tr>
<th>Number of absentees</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>&gt;6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency</td>
<td>8</td>
<td>15</td>
<td>11</td>
<td>8</td>
<td>3</td>
<td>4</td>
<td>1</td>
<td>0</td>
</tr>
</tbody>
</table>

(i) Illustrate these data by means of a vertical line chart.

**Ans:**

![Vertical line chart](image)

(ii) Calculate the mean.

<table>
<thead>
<tr>
<th>No. of absentee, $x$</th>
<th>Frequency $f$</th>
<th>$xf$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>8</td>
<td>8</td>
</tr>
<tr>
<td>1</td>
<td>15</td>
<td>15</td>
</tr>
<tr>
<td>2</td>
<td>11</td>
<td>22</td>
</tr>
<tr>
<td>3</td>
<td>8</td>
<td>24</td>
</tr>
<tr>
<td>4</td>
<td>3</td>
<td>12</td>
</tr>
<tr>
<td>5</td>
<td>4</td>
<td>20</td>
</tr>
<tr>
<td>6</td>
<td>1</td>
<td>6</td>
</tr>
</tbody>
</table>

$$\sum f = 50 \quad \sum xf = 99$$

$$Mean = \frac{\sum fx}{\sum f} = \frac{99}{50} = 1.98$$
Example: The times taken for 480 university students to travel from their accommodation to lectures are summarised below.

<table>
<thead>
<tr>
<th>Time (t minutes)</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 ≤ t &lt; 5</td>
<td>34</td>
</tr>
<tr>
<td>5 ≤ t &lt; 10</td>
<td>153</td>
</tr>
<tr>
<td>10 ≤ t &lt; 20</td>
<td>188</td>
</tr>
<tr>
<td>20 ≤ t &lt; 30</td>
<td>73</td>
</tr>
<tr>
<td>30 ≤ t &lt; 40</td>
<td>27</td>
</tr>
<tr>
<td>40 ≤ t &lt; 60</td>
<td>5</td>
</tr>
</tbody>
</table>

(i) Illustrate these data by means of a histogram.

Ans:

<table>
<thead>
<tr>
<th>Time (t minutes)</th>
<th>frequency</th>
<th>Class width</th>
<th>Frequency density</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 ≤ t ≤ 5</td>
<td>34</td>
<td>5</td>
<td>34 ÷ 5 = 6.8</td>
</tr>
<tr>
<td>5 ≤ t ≤ 10</td>
<td>153</td>
<td>5</td>
<td>153 ÷ 5 = 30.6</td>
</tr>
<tr>
<td>10 ≤ t ≤ 20</td>
<td>188</td>
<td>10</td>
<td>188 ÷ 10 = 18.8</td>
</tr>
<tr>
<td>20 ≤ t ≤ 30</td>
<td>73</td>
<td>10</td>
<td>73 ÷ 10 = 7.3</td>
</tr>
<tr>
<td>30 ≤ t ≤ 40</td>
<td>27</td>
<td>10</td>
<td>27 ÷ 10 = 2.7</td>
</tr>
<tr>
<td>40 ≤ t ≤ 60</td>
<td>5</td>
<td>20</td>
<td>5 ÷ 20 = 0.25</td>
</tr>
</tbody>
</table>

Example: The birth weights in grams of a random sample of 1000 babies are displayed in the cumulative frequency diagram below.
(i) Use the diagram to estimate the median and interquartile range of the data. [3]
Ans: Median = 3370, Q1=LQ=3050, Q3=UQ=3700, I-Q-R=650

**LESSON 5**

**Solving the Equations**

\[ \begin{align*}
- & + = - + \\
\text{All unknown to this side} & = \text{All known to this side}
\end{align*} \]

**Example:** Solve \( 3x - 8 = 25 \).

Ans: \( 3x - 8 = 25 \)
\( 3x = 25 + 8 \)
\( 3x = 33 \)
\( x = \frac{33}{3} = 11 \)

Check: \( 3x - 8 = 25 \) \hspace{1cm} (not needed in the exam)
\( 3 \times 11 - 8 = 25 \)
\( 33 - 8 = 25 \)
\( 25 = 25 \) \hspace{1cm} O.K

**Example:** Solve \( 2a + 4 = 6a - 4 \).

Ans: \( 2a + 4 = 6a - 4 \)
\( 2a - 6a = -4 - 4 \)
\( -4a = -8 \)
\( a = \frac{-8}{-4} = 2 \)
Check: \(2a + 4 = 6a - 4\)
\[2 \times 2 + 4 = 6 \times 2 - 4\]
\[4 + 4 = 12 - 4\]
\[8 = 8 \quad \text{O.K.}\]

**Example:** Solve \(2(x - 1) = 3(2x - 5) + 1\).

**Ans:** \(2(x - 1) = 3(2x - 5) + 1\)
\[2x - 2 = 6x - 15 + 1\]
\[2x - 6x = -15 + 1 + 2\]
\[-4x = -12\]
\[x = \frac{-12}{-4} = 3\]

**Check:** \(2(x - 1) = 3(2x - 5) + 1\)
\[2(3 - 1) = 3(2 \times 3 - 5) + 1\]
\[6 - 2 = 3(6 - 5) + 1\]
\[4 = 3 + 1\]
\[4 = 4 \quad \text{O.K.}\]

**Example:** Solve \(\frac{x + 1}{4} = \frac{4x - 7}{5}\). **Note:** If \(\frac{a}{b} = \frac{c}{d} \Rightarrow ad = bc\)

**Ans:** \(\frac{x + 1}{4} = \frac{4x - 7}{5}\)
\[4(4x - 7) = 5(x + 1)\]
\[16x - 28 = 5x + 5\]
\[16x - 5x = 28 + 5\]
\[11x = 33\]
\[x = \frac{33}{11} = 3\]

**Check:** \(\frac{x + 1}{4} = \frac{4x - 7}{5}\)
\[\frac{3 + 1}{4} = \frac{4 \times 3 - 7}{5}\]
\[\frac{4}{4} = \frac{12 - 7}{5}\]
\[4 = \frac{5}{5}\]
\[4 = 5\]
\[1 = 1 \quad \text{O.K.}\]
Example: Solve \( \frac{x+1}{4} - \frac{4x-7}{5} = 0 \).

\[
\frac{x+1}{4} = \frac{4x-7}{5} \\
4(4x-7) = 5(x+1) \\
16x - 28 = 5x + 5 \\
16x - 5x = 28 + 5 \\
11x = 33 \\
x = \frac{33}{11} = 3
\]

Example: Solve \( \frac{1}{4}x + \frac{1}{4} = \frac{4}{5}x - \frac{7}{5} \).

Ans:\[ \frac{1}{4}x - \frac{4}{5}x = -\frac{7}{5} - \frac{1}{4} \]

\[
\frac{5 - 16}{20}x = \frac{-28 - 5}{20} \\
\frac{-11}{20}x = \frac{-33}{20} \\
-11x = -33 \\
x = 3
\]

Check:\[ \frac{1}{4}x + \frac{1}{4} = \frac{4}{5}x - \frac{7}{5} \]

\[
\frac{3 + 1}{4} = \frac{4}{5} \times 3 - \frac{7}{5} \\
\frac{3+1}{4} = \frac{12}{5} - \frac{7}{5} \\
\frac{4}{4} = \frac{12}{5} - \frac{7}{5} \\
\frac{4}{4} = \frac{5}{4} - \frac{5}{5} \\
1 = 1 \quad \text{O.K} \]
Example: Solve \( \frac{6}{n+1} = 3 \).

Ans: \( \frac{6}{n+1} = 3 \)

\[
\frac{6}{n+1} = \frac{3}{1}
\]

\[3(n + 1) = 6\]

\[3n + 3 = 6\]

\[3n = 6 - 3\]

\[3n = 3\]

\[n = \frac{3}{3} = 1\]

Check: \( \frac{6}{n+1} = 3 \)

\[
\frac{6}{1+1} = \frac{3}{1}
\]

\[\frac{6}{2} = 3\]

\[3 = 3\quad \text{O.K}\]

Example: Solve \( 10 - \frac{4}{m} = 7 \).

Ans: \( -\frac{4}{m} = 7 - 10 \)

\[ -\frac{4}{m} = -3 \]

\[ -\frac{4}{m} = -\frac{3}{1} \]

\[ -3m = -4 \]

\[ 3m = 4 \]

\[ m = \frac{4}{3} \]
Check: \[10 - \frac{4}{m} = 7\]

\[10 - 4 \div m = 7\]
\[10 - 4 \div \frac{4}{3} = 7\]
\[10 - 4 \times \frac{3}{4} = 7\]
\[10 - 3 = 7\]
\[7 = 7\] O.K

Example: If I add 7 to my Mum’s age and then divide by 5. I get the same answer as if I subtract 2 from my Mum’s age and then divide by 4. Work out my Mum’s age.

Ans: Let Mum’s age = \(x\)

\[\frac{x + 7}{5} = \frac{x - 2}{4}\]

\[4(x + 7) = 5(x - 2)\]
\[4x + 28 = 5x - 10\]
\[4x - 5x = -10 - 28\]
\[-x = -38\]
\[x = 38\]

Check: \[\frac{38 + 7}{5} = \frac{38 - 2}{4}\]

\[\frac{45}{5} = \frac{36}{4}\]

\[9 = 9\] O.K
Example: Solve \(3(x + 1) = 2 + 4(2 - x)\).

**Ans:**

\[
egin{align*}
3x + 3 &= 2 + 8 - 4x \\
3x + 4x &= 2 + 8 - 3 \\
7x &= 7 \\
x &= \frac{7}{7} = 1
\end{align*}
\]

**Check:**

\[
egin{align*}
3(x + 1) &= 2 + 4(2 - x) \\
3(1 + 1) &= 2 + 4(2 - 1) \\
6 &= 2 + 8 - 4 \\
6 &= 6 \quad \text{O.K}
\end{align*}
\]

Example: Solve \(\frac{6}{x + 3} = \frac{9}{5 + 2x}\).

\[
egin{align*}
9(x + 3) &= 6(5 + 2x) \\
9x + 27 &= 30 + 12x \\
9x - 12x &= 30 - 27 \\
-3x &= 3 \\
x &= \frac{-3}{3} = -1
\end{align*}
\]

**Check:**

\[
\frac{6}{x + 3} = \frac{9}{5 + 2x}
\]

\[
\frac{6}{-1 + 3} = \frac{9}{5 - 2}
\]

\[
\frac{6}{2} = \frac{9}{3}
\]

\(3 = 3 \quad \text{O.K}\)
Example: \[
\frac{3}{x+2} + \frac{2}{x-2} = 0
\]
\[
\frac{3}{x+2} = -\frac{2}{x-2}
\]
\[
\frac{3}{x+2} = \frac{-2}{(x-2)} \Rightarrow 3(x-2) = -2(x+2)
\]
\[
3(x-2) = -2(x+2)
\]
\[
3x-6 = -2x-4
\]
\[
3x+2x = -4+6
\]
\[
5x = 2
\]
\[
x = \frac{2}{5} = 0.4
\]

OR \[
\frac{3}{x+2} = \frac{2}{(x-2)} \Rightarrow -3(x-2) = 2(x+2)
\]
\[
-3(x-2) = 2(x+2)
\]
\[
-3x+6 = 2x+4
\]
\[
-3x-2x = 4-6
\]
\[
-5x = -2
\]
\[
x = \frac{-2}{-5} = 0.4
\]

Example: \[
\frac{x+2}{3} - \frac{x-3}{2} = 1
\]
Ans: Multiply each side by 6
\[
6 \times \left( \frac{x+2}{3} - \frac{x-3}{2} \right) = 6 \times 1
\]
\[
2(x+2) - 3(x-3) = 6
\]
\[
2x+4 - 3x+9 = 6
\]
\[
x = 6 - 13
\]
\[
x = -7
\]
\[
x = 7
\]

Example: Find the value of \(x\) if the perimeter of the following triangle is \(21m\).

\[
\text{Ans: } P = \text{Perimeter} = x + x + 2 + x + 1 = 21
\]
\[
3x + 3 = 21
\]
\[
3x = 21 - 3 = 18 \Rightarrow x = 6
\]
Example: Find the value of $x$ if the area of the following triangle is $20m^2$.

\[ A = \frac{1}{2} b \times h = \frac{1}{2} (x + 2) \times 2 = 20 \]
\[ x + 2 = 20 \]
\[ x = 20 - 2 = 18 \]

Example: Find the value of $x$ if the perimeter of the following rectangle is $20m$.

\[ P = x + x + 2 + x + x + 2 = 20 \]
\[ 4x = 20 - 4 \]
\[ 4x = 16 \]
\[ x = 4 \]

Example: Find the value of $x$ if the perimeter of the following circle is $20m$.

\[ P = 2\pi r = 2\pi x = 20 \]
\[ x = \frac{20}{2\pi} = \frac{10}{\pi} \]

Example: Find the value of $x$.

\[ x + 2x + x + 2x = 360 \]
\[ 6x = 360 \]
\[ x = 60 \]

Example: Find the value of $x$.

\[ x + 2x = 180 \]
\[ 3x = 180 \]
\[ x = 60 \]
Example: Find the value of $x$.

\[
\text{Ans: } \frac{(n-2) \times 180}{n} = x
\]
\[
x = \frac{(6-2) \times 180}{6} = 120
\]

Example: The dimensions of a rectangle are shown below. Calculate the value of $x$.

\[
\text{Ans: } \begin{cases} 
    x + 2 = 2x - 1 & \text{for } x = 4 \\
    4x = 20 - 4 & \\
    x = 4 
\end{cases}
\]

Example: The dimensions of a right triangle are shown below. Calculate the value of $x$.

\[
\text{Ans: } \begin{cases} 
    (x - 1)^2 + (x - 2)^2 = x^2 & \\
    x^2 - 4x + 4 + x^2 - 2x + 1 = x^2 & \\
    x^2 - 6x + 5 = 0 & \\
    (x - 1)(x - 5) = 0 & \\
    \text{when } (x - 1) = 0 \Rightarrow x = 1 \Rightarrow \text{No good} & \\
    \text{when } (x - 5) = 0 \Rightarrow x = 5 \Rightarrow \text{OK} & 
\end{cases}
\]

Example: Solve $\sqrt{x + 1} = 3$.

\[
\text{Ans: } \left(\sqrt{x + 1}\right)^2 = 3^2
\]
\[
x + 1 = 9
\]
\[
x = 9 - 1
\]
\[
x = 8
\]
Example: Solve $|x| = 3$.
Ans: $x = \pm 3$

Example: Solve $|x - 1| = 3$.
Ans: $x - 1 = \pm 3$
(a) $x - 1 = 3 \Rightarrow x = 4$
(b) $x - 1 = -3 \Rightarrow x = -2$

Example: Solve $|2x + 1| = 4 - x$.
Ans: $2x + 1 = \pm (4 - x)$
(i) $2x + 1 = 4 - x$
$2x + x = 4 - 1$
$3x = 3$
$x = 1$
(ii) $2x + 1 = -(4 - x)$
$2x + 1 = -4 + x$
$2x - x = -4 - 1$
$x = -5$

Example: The square has the same perimeter as the triangle. This triangle is right-angled and has an area of $84 \text{ cm}^2$. Calculate the length, $d$, of the diagonal of the square. Give your answer correct to one decimal place.

Perimeter of triangle $= P = (2x + 1) + 2x + 7 = 4x + 8$
Perimeter of square $= P = y + y + y + y = 4y$
Area of triangle $= A = \frac{1}{2}(2x)(7) = 84$
$7x = 84$

Ans: $x = 12$
Example: Find three consecutive numbers with a sum of 15.

Ans:
The 3 numbers are: $x, (x+1), x+2$

$x + x + 1 + x + 2 = 15$
$3x + 3 = 15$

$3x = 15 - 3 = 12$
$x = \frac{12}{3} = 4$

The 3 numbers are: 4, 5, 6

Example: The angles of a triangles are $x, 2x and (3x+30)$. Find the value of $x$.

Ans:

$x + 2x + 3x + 30 = 180$
$6x = 180 - 30 = 150$

$x = \frac{150}{6} = 25$

The 3 angles are: 25, 50, 105

Example: In each case only one answer is correct. Ring the correct answer.

$3(4 - 5x) - x(2 - x)$ is equal to:

(a) $-x^2 - 17x + 12$  (b) $12 - 18x$  (c) $x^2 - 17x + 12$

(d) 2 or $\frac{4}{5}$  (e) $x^2 - 7x + 12$

Ans:

$3(4 - 5x) - x(2 - x)$
$12 - 15x - 2x + x^2$
$x^2 - 17x + 12$

Correct answer: (c) $x^2 - 17x + 12$
Example: The solution to the equation \(3(2x - 5) = 8\) is:

\[
\begin{align*}
(a) \quad \frac{13}{6} & \quad (b) \quad \frac{23}{6} & \quad (c) \quad -\frac{7}{6} \\
(d) \quad \frac{1}{6} & \quad (e) \quad 138
\end{align*}
\]

Ans: 
\[
\begin{align*}
3(2x - 5) &= 8 \\
6x - 15 &= 8 \\
6x &= 8 + 15 \\
x &= \frac{23}{6}
\end{align*}
\]
Correct answer: \(b\) \(\frac{23}{6}\)

Example: The outer rectangle shown in the Figure measures \((x + 2)\) by \((x + 1)\). The area of the un-shaded area is:

\[
\begin{align*}
(a) \quad 2x & \quad (b) \quad x^2 + 2x + 2 & \quad (c) \quad x^2 + 3x + 2 \\
(d) \quad x^2 + 5x + 2 & \quad (e) \quad x^2 + 3x + 2 - 2x
\end{align*}
\]
Correct answer: \(a\) \(2x\)

Example: The outer rectangle shown in the Figure measures \((2x + 3)\) by \((x + 2)\).

(a) Express the area of the shaded rectangle in terms of \(x\).

Ans: The area of the shaded rectangle = \(2x(x) = 2x^2\)
(b) Express the area of the unshaded rectangle in terms of $x$.

**Ans:** The area of the unshaded rectangle is:

$$\text{Area} = (x + 2)(2x + 3) - 2x^2 = 2x^2 + 3x + 4x + 6 - 2x^2 = 7x + 6$$

**OR**

![Diagram of rectangle and triangle](attachment:image.png)

Area = $A_1 + A_2 = 2(2x+3)+3x = 4x + 6 + 3x = 7x + 6$

(c) Write down the equation when the area of the shaded region is 2 square units less than the area of the unshaded area.

**Ans:** The area of the shaded rectangle = $2x^2$

The area of the unshaded rectangle = $7x + 6$

**Equation:** $\quad 2x^2 + 2 = 7x + 6$

**Example:** Jameela asked her uncle how old he was. “in 13 years, I’ll be twice as old as I was 7 years ago”, he replied.

**Ans:** Let her uncle’s age = $x$

$$x + 13 = 2(x - 7)$$

$$x + 13 = 2x - 14$$

$$x - 2x = -14 - 13$$

$$-x = -27$$

$$x = 27 \text{ old}$$

**Check:** In 13 years her uncle’s will be 27 + 13 = 40 years and 7 years ago he was 27 - 7 = 20 years, and therefore, 40 is twice of 20 and OK.

**Example:** The rectangle and the triangle have the same area. Write and solve an equation for $x$ and hence find the area of these shapes.
Ans: \[ \text{Area of triangle} = \frac{1}{2}(x + 5) \times 3 = \frac{3(x + 5)}{2} \]
\[ \text{Area of square} = 4x \]
\[ \text{Area of triangle} = \text{Area of the square} \]
\[ \frac{3(x + 5)}{2} = 4x \]
\[ \frac{3(x + 5)}{2} = \frac{4x}{1} \]
\[ 3(x + 5) = 2(4x) \]
\[ 3x + 15 = 8x \]
\[ 3x - 8x = -15 \]
\[ -5x = -15 \]
\[ x = \frac{-15}{-5} = 3 \]
\[ \text{Area of triangle} = \frac{3(x + 5)}{2} = \frac{3(3 + 5)}{2} = \frac{24}{2} = 12 \]
\[ \text{Area of square} = 4x = 4 \times 3 = 12 \]

Check: \[ \text{Area of triangle} = 12 \]
\[ \text{Area of square} = 12 \]
\[ 12 = 12 \quad \text{O.K} \]

Example: You think of a number, divide it by 3, and add 10. The result is 16. What is the original number?

Ans: Let \( x \) be the number. Then \( \frac{x}{3} + 10 = 16 \)

Equation: \[ \frac{x}{3} + 10 = 16 \]
\[ \frac{x}{3} = 16 - 10 \]
\[
\frac{x}{3} = 6 \\
x = 6 \times 3 \\
x = 18
\]

Check:
\[
\frac{x}{3} + 10 = 16 \\
\frac{18}{3} + 10 = 16 \\
6 + 10 = 16
\]

\[16 = 16 \quad \text{O.K}\]

**Example:** Two years ago Omar’s father was exactly four times his age, and their ages added up to 50. How old is Omar now?

**Ans:** Let \( x \) be Omar’s present age.
Then his age 2 years ago was \((x-2)\)
Then his father age 2 years ago was \(4(x-2)\)

**Equation:**
\[
\text{Omar’s age 2 years ago} + \text{his father’s age 2 years ago} = 50 \\
(x-2) + 4(x-2) = 50
\]
\[
x - 2 + 4x - 8 = 50 \\
5x = 50 + 10 \\
5x = 60 \\
x = \frac{60}{5} \\
x = 12
\]

Omar’s present age = 12

Check:

Omar’s age 2 years ago was: \( 12 - 2 = 10 \)
His father’s age 2 years ago was: \(4 \times 10 = 40 \)

And \(10 + 40 = 50 \): O.K

**Example:** A man is 8 times as old as his son. In five years time he will be only three times as old. How old is the son now?

**Ans:**
Let present age of son is \( x \)
Then present age of his father is \( 8x \)
In 5 year time, the age of son is $x + 5$
In 5 year time, the age of father is $8x + 5$

Equation:

$$8x + 5 = 3(x + 5)$$
$$8x + 5 = 3x + 15$$
$$8x - 3x = 15 - 5$$
$$5x = 10$$
$$x = \frac{10}{5} = 2$$

Son = 2 years old
Father = 16 years old

In 5 year time, the age of son is $2 + 5 = 7$
In 5 year time, the age of father is $16 + 5 = 21$
And $21 = 3 \times 7$ O.K

Example: The sum of the external angles of a polygon is $360^\circ$.

(a) Write an algebraic expression for the size of an external angle of a regular $n$-sided polygon.

(b) By forming and solving an equation, find the number of sides of a regular polygon with an external angle of $72^\circ$

Ans: (a) External angle of a regular $n$-sided polygon $= \frac{360^\circ}{n}$.

(b) Sides of a regular polygon $n = \frac{360^\circ}{\text{external angle}} = \frac{360}{72} = 5$.

Look at the following for information
Note: Angles \(a, b, c, d, e\) are exterior angle and their sum = 360

Note: Interior angle + exterior angle = 180, i.e. \(c + f = 180\)

LESSON 6

Simultaneous Equations

Example: Solve these simultaneous equations.

\[
\begin{align*}
8x + 9y &= 60 \quad \text{...(1)} \\
3x + 4y &= 25 \quad \text{...(2)}
\end{align*}
\]

Multiply each side of Eq.1 by 3 and each side of Eq.2 by 8

\[
\begin{align*}
24x + 27y &= 180 \quad \text{...(1)} \\
24x + 32y &= 200 \quad \text{...(2)}
\end{align*}
\]

Subtract Eq.2 from Eq.1

\[
\begin{align*}
-5y &= -20 \\
y &= \frac{-20}{-5} = 4
\end{align*}
\]

Now substitute the value of \(y = 4\) into either Eq.1 or Eq.2

\[
\begin{align*}
24x + 27y &= 180 \quad \text{...(1)} \\
24x + 27 \times 4 &= 180 \quad \text{...(1)} \\
24x &= 180 - 108 = 72 \quad \text{...(1)} \\
x &= \frac{72}{24} = 3 \quad \text{...(1)}
\end{align*}
\]
Check:
\[
\begin{align*}
\frac{2}{3}x + \frac{3}{4}y &= 5 \\
\frac{2}{3} \times 3 + \frac{3}{4} \times 4 &= 5 \\
2 + 3 &= 5 \quad \text{OK}
\end{align*}
\]

Example: Solve the following simultaneous equations.
\[
\begin{align*}
\frac{1}{3}x + \frac{1}{5}y &= 2 \\
\frac{2}{3}x + \frac{3}{5}y &= 5
\end{align*}
\]

Ans:
\[
\begin{align*}
\frac{1}{3}x + \frac{1}{5}y &= 2 \quad \text{...(1)} \\
\frac{2}{3}x + \frac{3}{5}y &= 5 \quad \text{...(2)}
\end{align*}
\]

Multiply each side of Eq.1 and 2 by 15
\[
\begin{align*}
15 \times \left( \frac{1}{3}x + \frac{1}{5}y \right) &= 2 \times 15 \quad \text{...(1)} \\
15 \times \left( \frac{2}{3}x + \frac{3}{5}y \right) &= 5 \times 15 \quad \text{...(2)}
\end{align*}
\]

\[
\begin{align*}
5x + 3y &= 30 \quad \text{...(1)} \\
10x + 9y &= 75 \quad \text{...(2)}
\end{align*}
\]

Multiply each side of Eq.1 by 2
\[
\begin{align*}
10x + 6y &= 60 \quad \text{...(1)} \\
10x + 9y &= 75 \quad \text{...(2)}
\end{align*}
\]

Subtract Eq.2 from Eq.1
\[
\begin{align*}
-3y &= -15 \quad \text{...(3)} \\
y &= \frac{-15}{-3} = 5 \quad \text{...(3)}
\end{align*}
\]
Now substitute the value of \( y = 5 \) into either Eq.1 or Eq.2

\[
10x + 6y = 60 \quad \text{...(1)}
\]
\[
10x + 6 \times 5 = 60 \quad \text{...(1)}
\]
\[
10x = 60 - 30 = 30 \quad \text{...(1)}
\]
\[
x = \frac{30}{10} = 3 \quad \text{...(1)}
\]

**Check:**

\[
3x - 5y = 15 \quad \text{...(1)}
\]
\[
3 \times 2.5 - 5 \times (-1.5) = 15 \quad \text{...(1)}
\]
\[
7.5 + 7.5 = 15 \quad \text{...(1)}
\]
\[
15 = 15 \quad \text{O.K} \quad \text{...(1)}
\]

**Example:** Solve the following simultaneous equations.

\[
-\frac{1}{3}x + \frac{1}{5}y = 2 \quad \text{...(1)}
\]
\[
\frac{2}{3}x + \frac{3}{5}y = 5 \quad \text{...(2)}
\]

**Ans:**

\[
-\frac{1}{3}x + \frac{1}{5}y = 2 \quad \text{...(1)}
\]
\[
\frac{2}{3}x + \frac{3}{5}y = 5 \quad \text{...(2)}
\]

Multiply each side of Eq.1 and 2 by 15

\[
15 \left( -\frac{1}{3}x + \frac{1}{5}y \right) = 2 \times 15 \quad \text{...(1)}
\]
\[
15 \left( \frac{2}{3}x + \frac{3}{5}y \right) = 5 \times 15 \quad \text{...(2)}
\]
\[
-5x + 3y = 30 \quad \text{...(1)}
\]
\[
10x + 9y = 75 \quad \text{...(2)}
\]

Multiply each side of Eq.1 by 2

\[
-10x + 6y = 60 \quad \text{...(1)}
\]
\[
10x + 9y = 75 \quad \text{...(2)}
\]
Add Eq.2 and Eq.1

\[ 15y = 135 \]  
\[ y = \frac{135}{15} = 9 \]  

...(3)

Now substitute the value of \( y = 9 \) into either Eq.1 or Eq.2

\[ -10x + 6y = 60 \]  
\[ -10x + 6 \times 9 = 60 \]  
\[ -10x = 60 - 54 = 6 \]  
\[ x = \frac{6}{-10} = -0.6 \]  

...(1)

Check:

\[ -5x + 3y = 30 \]  
\[ -5 \times (-0.6) + 3 \times (9) = 30 \]  
\[ 3 + 27 = 30 \]  
\[ 30 = 30 \text{ O.K} \]  

...(1)

**LESSON 7**

**Graphical Solution of Simultaneous Equations**

Simultaneous equations can be solved by using graphs to plot the two equations. The coordinates of the point of intersection gives the solution of the simultaneous Equations.

**Example:** Solve these simultaneous equations using graphs.

\[ y = x + 4 \]
\[ y = 2x + 3 \]

**Ans:** The two equations are plotted as straight line and their intersection is at point A(1, 3) and that is the solution of the two simultaneous equations, i.e. \( x = 1 \) and \( y = 3 \).
Example: By drawing suitable graphs, solve the following pairs of simultaneous equations.

\[ x - y + 4 = 0 \Rightarrow -y = -x - 4 \Rightarrow y = x + 4 \]  
\[ 4x - y - 2 = 0 \Rightarrow -y = -4x + 2 \Rightarrow y = 4x - 2 \]  
\[ \text{...(Eq.1)} \]
\[ \text{...(Eq.2)} \]

Ans: The two equations are plotted as straight line and their intersection is at point A(2, 6) and that is the solution of the two simultaneous equations, i.e. \( x = 2 \) and \( y = 6 \).
Example: By drawing suitable graphs, solve the following pairs of simultaneous equations.

\[ \begin{align*}
  x - y &= 1 \Rightarrow -y = -x + 1 \quad \Rightarrow y = x - 1 & \quad \text{...(Eq.1)} \\
  2x + y + 7 &= 0 \quad \Rightarrow y = -2x - 7 & \quad \text{...(Eq.2)}
\end{align*} \]

Ans: The two equations are plotted as straight line and their intersection is at point \( A(-2, -3) \) and that is the solution of the two simultaneous equations, i.e. \( x = -2 \) and \( y = -3 \).

Example: Solve these simultaneous equations.

\[ \begin{align*}
  x + 2y &= 11 & \quad \text{...(Eq.1)} \\
  2x - y &= -3 & \quad \text{...(Eq.2)}
\end{align*} \]

Ans: The above simultaneous equations can be solved in three ways:

(a) Substituting the value of \( x \) or \( y \) from Eq.1 into Eq.2 or vice versa.

\[ \begin{align*}
  x + 2y &= 11 & \quad \text{...(Eq.1)} \\
  2x - y &= -3 & \quad \text{...(Eq.2)}
\end{align*} \]

Lets substitute the value of \( x \) from Eq.1 into Eq.2.

\[ \begin{align*}
  x &= 11 - 2y & \quad \text{(Eq.1)} \\
  2(11 - 2y) - y &= -3 & \quad \text{(Eq.2)}
\end{align*} \]

\[ \begin{align*}
  22 - 4y - y &= -3 & \quad \text{(Eq.3)} \\
  -5y &= -3 - 22 & \quad \text{(Eq.3)}
\end{align*} \]
\[-5y = -25 \quad \text{(Eq. 3)}
\]
\[y = \frac{-25}{-5} = 5 \quad \text{(Eq. 3)}
\]
\[x = 11 - 2y = 11 - 2 \times 5 = 11 - 10 = 1 \quad \text{(Eq. 1)}
\]
\[\Rightarrow x = 1
\]

(b) Making the coefficients of \(x\) or \(y\) equal and then subtract one equation from the other.
\[
x + 2y = 11 \quad \text{...(Eq. 1)}
\]
\[2x - y = -3 \quad \text{...(Eq. 2)}
\]
If both sides of Eq. 1 is multiplied by 2, then
\[
2x + 4y = 22 \quad \text{...(Eq. 1)}
\]
\[2x - y = -3 \quad \text{...(Eq. 2)}
\]
If Eq. 2 is subtracted from Eq. 2 as follows:
\[
5y = 25
\]
\[y = 5
\]
\[x + 2y = 11 \quad \text{...(Eq. 1)}
\]
\[x + 2 \times 5 = 11 \quad \text{...(Eq. 1)}
\]
\[x = 11 - 10 \quad \text{...(Eq. 1)}
\]
\[x = 1
\]

**LESSON 8**

**Solution of Simultaneous Equations by determinants**

**Example:** Solve these simultaneous equations.
\[
x + 2y = 11 \quad \text{...(Eq. 1)}
\]
\[2x - y = -3 \quad \text{...(Eq. 2)}
\]
By the method of determinant as follows:
\[
x + 2y = 11 \quad \text{...(Eq. 1)}
\]
\[2x - y = -3 \quad \text{...(Eq. 2)}
\]
\[
\Delta = \begin{vmatrix} 1 & 2 \\ 2 & -1 \end{vmatrix} = 1 \times (-1) - [2 \times 2] = -1 - 4 = -5
\]
\[
\Delta_x = \begin{vmatrix} 11 & 2 \\ -3 & -1 \end{vmatrix} = 11 \times (-1) - [2 \times (-3)] = -11 + 6 = -5
\]
Following is the description of how the two-unknown equations can be solved.

\[ a_1x + b_1y = c_1 \] ... (1)
\[ a_2x + b_2y = c_2 \] ... (2)

\[ \Delta = \begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix} = a_1 \times b_2 - b_1 \times a_2 \]

\[ \Delta_x = \begin{vmatrix} c_1 & b_1 \\ c_2 & b_2 \end{vmatrix} = c_1 \times b_2 - b_1 \times c_2 \]

\[ \Delta_y = \begin{vmatrix} a_1 & c_1 \\ a_2 & c_2 \end{vmatrix} = a_1 \times c_2 - c_1 \times a_2 \]

\[ x = \frac{\Delta_x}{\Delta} \quad y = \frac{\Delta_y}{\Delta} \]

**Example:** Consider the following equations:

\[ 2x + 3y = 18 \] ... (1)
\[ 5x - 2y = 7 \] ... (2)

\[ \Delta = \begin{vmatrix} 2 & 3 \\ 5 & -2 \end{vmatrix} = 2 \times (-2) - [3 \times 5] = -4 - 15 = -19 \]

\[ \Delta_x = \begin{vmatrix} 18 & 3 \\ 7 & -2 \end{vmatrix} = 18 \times (-2) - [3 \times 7] = -36 - 21 = -57 \]

\[ \Delta_y = \begin{vmatrix} 2 & 18 \\ 5 & 7 \end{vmatrix} = 2 \times 7 - [18 \times 5] = 14 - 90 = -76 \]

\[ x = \frac{\Delta_x}{\Delta} = \frac{-57}{-19} = 3 \quad y = \frac{\Delta_y}{\Delta} = \frac{-76}{-19} = 4 \]
**Example:** Consider the following equations:

\[-7x + 4y = -5 \quad \text{...(1)}\]
\[9x - 5y = 7 \quad \text{...(2)}\]

\[\Delta = \begin{vmatrix} -7 & 4 \\ 9 & -5 \end{vmatrix} = (-7) \times (-5) - [4 \times 9] = 35 - 36 = -1\]

\[\Delta_x = \begin{vmatrix} -5 & 4 \\ 7 & -5 \end{vmatrix} = (-5) \times (-5) - [4 \times 7] = 25 - 28 = -3\]

\[\Delta_y = \begin{vmatrix} -7 & -5 \\ 9 & 7 \end{vmatrix} = (-7) \times 7 - [9 \times (-5)] = -49 - (-45) = -49 + 45 = -4\]

\[x = \frac{\Delta_x}{\Delta} = \frac{-3}{-1} = 3 \quad y = \frac{\Delta_y}{\Delta} = \frac{-4}{-1} = 4\]

Look at the following method for solving the given equation:

\[
\Delta = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = a_1b_2c_3 + b_1c_2a_3 + c_1a_2b_3 - [a_3b_2c_1 + b_3c_2a_1 + c_3a_2b_1]
\]

\[
\Delta_x = \begin{vmatrix} d_1 & b_1 & c_1 \\ d_2 & b_2 & c_2 \\ d_3 & b_3 & c_3 \end{vmatrix} = d_1b_2c_3 + b_1c_2d_3 + c_1d_2b_3 - [d_3b_2c_1 + b_3c_2d_1 + c_3d_2b_1]
\]

\[
\Delta_y = \begin{vmatrix} a_1 & d_1 & c_2 \\ a_2 & d_2 & c_2 \\ a_3 & d_3 & c_3 \end{vmatrix} = a_1d_2c_3 + d_1c_2a_3 + c_1a_2d_3 - [a_3d_2c_1 + d_3c_2a_1 + c_3a_2d_1]
\]

\[
\Delta_z = \begin{vmatrix} a_1 & b_1 & d_1 \\ a_2 & b_2 & d_2 \\ a_3 & b_3 & d_3 \end{vmatrix} = a_1b_2d_3 + b_1d_2a_3 + d_1a_2b_3 - [a_3b_2d_1 + b_3d_2a_1 + d_3a_2b_1]
\]
Consider the following example

\[ \begin{align*}
x + y + z &= 6 \quad \text{...(1)} \\
2x - y + 3z &= 9 \quad \text{...(2)} \\
-x + 3y - 2z &= -1 \quad \text{...(3)}
\end{align*} \]

\[
\Delta = \begin{vmatrix} 1 & 1 & 1 \\ 2 & -1 & 3 \\ -1 & 3 & -2 \end{vmatrix} = 2 - 1 \cdot 3 - 2 = -1 = 2 - 3 + 6 - [1 + 9 - 4] = 5 - 6 = -1
\]

\[
x = \frac{\Delta_x}{\Delta} = \frac{1 \cdot 1 \cdot 1 - 1 \cdot 3 \cdot -1}{-1} = \frac{12 - 3 + 27 - [1 + 54 - 18]}{-1} = \frac{36 - 37}{-1} = 1
\]

\[
y = \frac{\Delta_y}{\Delta} = \frac{-1 \cdot 1 \cdot 1 - 6 \cdot 3 \cdot -1}{-1} = \frac{-18 - 18 - 2 - [9 - 3 - 24]}{-1} = \frac{-38 + 36}{-1} = 2
\]

\[
z = \frac{\Delta_z}{\Delta} = \frac{-1 \cdot 1 \cdot 6 - 1 \cdot 2 \cdot -1}{-1} = \frac{1 - 9 + 36 - [6 + 27 - 2]}{-1} = \frac{28 - 31}{-1} = 3
\]
LESSON 9

Cartesian Coordinates System

- **y-axis**
- **x-axis**
- **Origin (0, 0)**
- **Coordinates of point P**

- **Quadrant I**
  - $x = +$
  - $y = +$

- **Quadrant II**
  - $x = -$  
  - $y = +$

- **Quadrant III**
  - $x = -$  
  - $y = -$

- **Quadrant IV**
  - $x = +$  
  - $y = -$
**Example:** Plot the following points on the graph below.

\[ P_1(10,10) \quad P_2(30,20) \quad P_3(-10,10) \quad P_4(-22,-14) \quad P_5(32,-18) \quad P_6(10,-10) \]

**Equation of a straight line**

The standard form of a straight line equation is of the form:  
\[ y = mx + c \]

Where \( m \) is the gradient (slope) and \( c \) is the y-intercept.

The gradient \( m = \frac{y_2 - y_1}{x_2 - x_1} \) and it does not matter which point is taken as \( P_1 \) and \( P_2 \) as it can be seen as follow:
\[
m_1 = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3 - 7}{1 - 3} = \frac{-4}{-2} = 2
\]

\[
m_2 = \frac{y_2 - y_1}{x_2 - x_1} = \frac{7 - 3}{3 - 1} = \frac{4}{2} = 2
\]
Note: The gradients of lines 1 and 3 are positive and the gradients of lines 2 and 4 are negatives.

Consider the following graph. There are equations for various straight lines. These straight line equations are as follows:

1. \( y = mx \) (Black Colour)
2. \( y = -mx \) (Red Colour)
3. \( y = 3 \) (Green Colour)
4. \( y = -3 \) (Light Blue Colour)
5. \( x = 3 \) (Blue Colour)
6. \( x = -3 \) (Pink Colour)
Example: Find the equation of the line that passes through the points (-3,5) and (5,-7).

First, find the slope: Where \( m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{5 - (-7)}{-3 - (-5)} = \frac{5 + 7}{-3 + 5} = \frac{12}{2} = 6 \)

\( y = mx + c = 6x + c \) \hspace{1cm} 5 = 6 \times (-3) + c \Rightarrow c = 5 + 18 = 23 \hspace{1cm} y = 6x + 23 \) is the equation of the line.
Example: Find the gradient and the y-intercept of $y = 3x + 7$.

Ans: Gradient $= m = 3$ and y-intercept $= c = 7$
Example: Find the gradient and the y-intercept of \(-y = 3x + 7\).

Ans: 
\[-y = 3x + 7\]
\[y = -3x - 7\]
Gradient \(= m = -3\) and y-intercept \(= c = -7\)

Example: Find the gradient and the y-intercept of \(2y = 3x + 7\).

Ans: 
\[2y = 3x + 7\]
\[y = \frac{3}{2}x + \frac{7}{2}\]

Example: \(\text{Gradient } m = \frac{3}{2} \text{ and } \text{y-intercept } c = \frac{7}{2}\)

Find the gradient and the y-intercept of \(2y - 3x - 7 = 0\).

Ans: 
\[2y - 3x - 7 = 0\]
\[2y = 3x + 7\]
\[y = \frac{3}{2}x + \frac{7}{2}\]
Gradient \(= m = \frac{3}{2}\) and y-intercept \(= c = \frac{7}{2}\)

Example: Find the gradient and the y-intercept of \(\frac{1}{2}y - 3x - 7 = 0\).

Ans: 
\[\frac{1}{2}y - 3x - 7 = 0\]
\[\frac{1}{2}y = 3x + 7\]
\[y = 2(3x + 7)\]
\[y = 6x + 14\]
Gradient \(= m = 6\) and y-intercept \(= c = 14\)

Example: Find the gradient and the y-intercept of \(-3x - 7 = -2y\).

Ans: 
\[-3x - 7 = -2y\]
\[2y = 3x + 7\]
\[y = \frac{3}{2}x + \frac{7}{2}\]
Gradient \(= m = \frac{3}{2}\) and y-intercept \(= c = \frac{7}{2}\)

Example: Find the gradient and the y-intercept of \(\frac{3}{5}y - \frac{3}{4}x - 7 = 0\).

Ans: 
\[\frac{3}{5}y - \frac{3}{4}x - 7 = 0\]
\[
\frac{3}{5} y = \frac{3}{4} x + 7 \\
\frac{5}{3} \left( \frac{3}{5} y \right) = \left( \frac{5}{3} \right) \left( \frac{3}{4} x + 7 \right) \\
y = \frac{15}{12} x + \frac{35}{3}
\]

Gradient \( m = \frac{15}{12} \) and y-intercept \( c = \frac{35}{3} \)

**Example:** Find the gradient and the y-intercept of \( -\frac{2}{5} y + \frac{3}{7} x - 7 = 0 \).

**Ans:** \( -\frac{2}{5} y + \frac{3}{7} x - 7 = 0 \)

\[
-\frac{2}{5} y = -\frac{3}{7} x + 7 \\
\frac{5}{2} \left( -\frac{2}{5} y \right) = \left( \frac{5}{2} \right) \left( -\frac{3}{7} x + 7 \right) \\
- y = -\frac{15}{14} x + \frac{35}{2} \\
y = \frac{15}{14} x - \frac{35}{2}
\]

Gradient \( m = \frac{15}{14} \) and y-intercept \( c = -\frac{35}{2} \)

**Example:** Find the equation of a line with gradient of 3 and y-intercept of -5.

**Ans:** \( y = mx + c \)
\( y = 3x - 5 \)
Example: Find the equation of a line with gradient of 3 and passes through point \( A(0, 5) \).

\[
y = mx + c \\
y = 3x + c \\
5 = 3 \times 0 + c \\
c = 5 \\
y = 3x + 5
\]

Example: Find the equation of a line which passes through points \( A(0, 5) \) and \( B(2, 11) \).

\[
m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{11 - 5}{2 - 0} = \frac{6}{2} = 3 \\
y = 3x + c \\
5 = 3 \times 0 + c \\
c = 5 \Rightarrow y = 3x + 5
\]

Example: Find the equation of a line which passes through points \( A(-2, -13) \) and \( B(1, -4) \).

\[
m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-4 - (-13)}{1 - (-2)} = \frac{-4 + 13}{1 + 2} = \frac{9}{3} = 3 \\
y = 3x + c \\
-4 = 3 \times 1 + c \\
c = -4 - 3 \\
c = -7 \\
y = 3x - 7
\]

Example: Find the gradient and the \( y \)-intercept of the following straight lines.

(a) \( y = -4x + 17 \)

\[
\text{Ans:} \quad y = mx + c \\
\Rightarrow m = -4 \quad \text{and} \quad c = 17
\]

(b) \( 2y = -4x + 16 \)

\[
\text{Ans:} \quad y = -\frac{4}{2}x + \frac{16}{2} \\
y = -2x + 8 \\
\Rightarrow m = -2 \quad \text{and} \quad c = 8
\]
Gradients of Parallel Lines

\[ \text{Gradient of } L_1: \quad m_1 = \frac{y_2 - y_1}{x_2 - x_1} = \frac{7 - 0}{-2 - (-9)} = \frac{7}{7} = 1 \]

\[ \text{Gradient of } L_2: \quad m_2 = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4 - 0}{0 - (-4)} = \frac{4}{4} = 1 \]

\[ \text{Gradient of } L_3: \quad m_3 = \frac{y_2 - y_1}{x_2 - x_1} = \frac{0 - (-4)}{4 - 0} = \frac{4}{4} = 1 \]

\[ \text{Gradient of } L_4: \quad m_4 = \frac{y_2 - y_1}{x_2 - x_1} = \frac{0 - (-7)}{7 - 0} = \frac{7}{7} = 1 \]

\[ \text{Gradient of } L_5: \quad m_5 = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3 - 6}{8 - 5} = \frac{-3}{3} = -1 \]

\[ \text{Gradient of } L_6: \quad m_6 = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1 - 6}{6 - 1} = \frac{-5}{5} = -1 \]

\[ \text{Gradient of } L_7: \quad m_7 = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-2 - (-5)}{-5 - (-2)} = \frac{-2 + 5}{-5 + 2} = \frac{3}{-3} = -1 \]

\[ \text{Gradient of } L_8: \quad m_8 = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-2 - (-7)}{-9 - (-4)} = \frac{-2 + 7}{-9 + 4} = \frac{5}{-5} = -1 \]

**Note:** When two straight lines are parallel, then their gradients are the same. For example, lines \( L_1, L_2, L_3, L_4 \) are parallel with each other, then their gradients:
Also, lines $L_5$, $L_6$, $L_7$, $L_8$ are parallel with each other, then their gradients: $m_5 = m_6 = m_7 = m_8 = -1$.

**Gradients of Perpendicular Lines**

![Graph showing perpendicular lines](image)

**Gradient of $L_1$:**

$m_1 = \frac{y_2 - y_1}{x_2 - x_1} = \frac{7 - 0}{-4 - (-6)} = \frac{7}{2}$

**Gradient of $L_2$:**

$m_2 = \frac{y_2 - y_1}{x_2 - x_1} = \frac{7 - 5}{-4 - 3} = \frac{2}{-7}$

**Gradient of $L_3$:**

$m_3 = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-3 - (-7)}{4 - (-5)} = \frac{4}{9}$

**Gradient of $L_4$:**

$m_4 = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-7 - 2}{-5 - (-9)} = \frac{-9}{4}$

**Note:** When two straight lines are perpendicular, then their gradients are: $m_1 = - \frac{1}{m_2}$, or $m_2 = - \frac{1}{m_1}$. For example, lines $L_1$ and $L_2$ are perpendicular with each other, then their gradients: $m_1 = \frac{7}{2}$ and $m_2 = - \frac{2}{7}$, i.e. $m_1 = - \frac{1}{m_2}$. Also, lines
$L_3$ and $L_4$ are perpendicular with each other, then their gradients: \( m_3 = \frac{4}{9} \) and
\[ m_4 = -\frac{9}{4}, \text{ i.e. } m_3 = -\frac{1}{m_4}. \]

**Example:** Find the equation of a straight line which is parallel to \( y = 3x + 7 \) and passes through point (2, -5).

**Ans:**
\[ y = mx + c \\
y = 3x + c \\
-5 = 3 \times 2 + c \\
-5 = 6 + c \\
c = -5 - 6 = -11 \\
y = 3x - 11 \text{ is the equation of that line.} \]

**Example:** Find the equation of a straight line which is parallel to \( 2y = 3x + 7 \) and passes through point (2, -5).

**Ans:**
\[ 2y = 3x + 7 \\
y = \frac{3}{2}x + \frac{7}{2} \\
y = mx + c \\
y = 3x + c \\
-5 = \frac{3}{2} \times 2 + c \\
-5 = 3 + c \\
c = -5 - 3 = -8 \\
y = \frac{3}{2}x - 8 \text{ is the equation of that line.} \]

**Example:** Find the equation of a straight line which is parallel to \( 2y = \frac{3}{4}x + 7 \) and passes through point (-2, 5).

**Ans:**
\[ 2y = \frac{3}{4}x + 7 \\
y = \frac{3}{8}x + \frac{7}{2} \\
y = mx + c \\
y = \frac{3}{8}x + c \\
5 = \frac{3}{8} \times (-2) + c \\
5 = -\frac{6}{8} + c \]
\[ c = 5 + \frac{6}{8} = \frac{40}{8} + \frac{6}{8} = \frac{46}{8} \]

\[ y = \frac{3}{8}x + \frac{46}{8} \] is the equation of that line.

**Example:** Find the equation of a straight line which is perpendicular to \( y = 3x + 7 \) and passes through point \((2, -5)\).

**Ans:** \( y = mx + c \)

\[ m = -\frac{1}{3} \]

\[ y = -\frac{1}{3}x + c \]

\[-5 = -\frac{1}{3} \times 2 + c \]

\[-5 = -\frac{2}{3} + c \]

\[ c = -5 + \frac{2}{3} = -\frac{15}{3} + \frac{2}{3} = -\frac{13}{3} \]

\[ y = -\frac{1}{3}x - \frac{13}{3} \] is the equation of that line.

**Example:** Find the equation of a straight line which is perpendicular to \( 2y = 3x + 7 \) and passes through point \((-2, 3)\).

\[ 2y = 3x + 7 \]

\[ y = \frac{3}{2}x + \frac{7}{2} \]

\[ m = -\frac{2}{3} \]

\[ y = mx + c \]

\[ y = -\frac{2}{3}x + c \]

\[ 3 = -\frac{2}{3} \times (-2) + c \]

\[ 3 = \frac{4}{3} + c \]

\[ c = 3 - \frac{4}{3} = \frac{9}{3} - \frac{4}{3} = \frac{5}{3} \]

\[ y = -\frac{2}{3}x + \frac{5}{3} \] is the equation of that line.
Example: Find the equation of a straight line which is perpendicular to \(2y = \frac{3}{4}x + 7\) and passes through point \((-2, 5)\).

Ans: 

\[
2y = \frac{3}{4}x + 7
\]

\[
y = \frac{3}{8}x + \frac{7}{2}
\]

\[
m = -\frac{8}{3}
\]

\[
y = mx + c
\]

\[
y = -\frac{8}{3}x + c
\]

\[
5 = -\frac{8}{3}(-2) + c
\]

\[
5 = \frac{16}{3} + c
\]

\[
c = 5 - \frac{16}{3} = \frac{15}{3} - \frac{16}{3} = -\frac{1}{3}
\]

\[
y = -\frac{8}{3}x - \frac{1}{3}
\]

is the equation of that line.
**LESSON 10**

**Plotting The Graphs**

**Examples:**

Plot the graphs of the following straight lines.

(a) \( y = x + 4 \)  
(b) \( y = 4 - \frac{2}{3}x \)
(c) \( y = -3x + 6 \)  
(d) \( y = 3x - 6 \)

**Solution**

(a) \( y = x + 4 \)

\[
\begin{array}{c|c|c}
  y & x & \text{OR} \\
--- & --- & --- \\
  4 & 0 & 4 & 0 \\
  0 & -4 & 8 & 4 \\
\end{array}
\]

\( P_1(0, 4) \)  
\( P_2(-4, 0) \)  
\( P_3(4, 8) \)
(b) \( y = 4 - \frac{2}{3}x \)
\( y = -\frac{2}{3}x + 4 \)

<table>
<thead>
<tr>
<th>( y )</th>
<th>( x )</th>
<th>OR</th>
<th>( y )</th>
<th>( x )</th>
</tr>
</thead>
<tbody>
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<td>4</td>
<td>0</td>
<td></td>
<td>4</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>6</td>
<td></td>
<td>2</td>
<td>3</td>
</tr>
</tbody>
</table>

\( P_1(0, 4) \) and \( P_2(6, 0) \) OR \( P_1(0, 4) \) and \( P_3(3, 2) \)
(c) \( y = 3x - 6 \)

\[
\begin{array}{c|c|c|c|c}
\text{y} & \text{x} & \text{OR} & \text{y} & \text{x} \\
-6 & 0 & -6 & 0 \\
0 & 2 & 3 & 3 \\
\end{array}
\]

\[ P_1(0, -6) \text{ and } P_2(2, 0) \text{ OR } P_1(0, -6) \text{ and } P_3(3, 3) \]
Example:

Find the equation of a straight line which passes through points $P_1$ and $P_2$ as shown in Fig.1.

![Figure 1](image)

Solution

The standard equation of a straight line is: $y = mx + c$, so we need to find the gradient, $m$ and the y-intercept, $b$ as follows:

$P_1(1, 6)$ and $P_2(-3, 2)$

$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2 - 6}{-3 - 1} = \frac{-4}{-4} = 1$

$y = mx + b$

$y = x + b$

$6 = 1 + b$

$b = 6 - 1$

$b = 5$

$y = x + 5$
Example:

Find the equation of a straight line as shown in Fig.2
Solution

Looking at the graph, the line crosses the y axis at (0, 5), and therefore, the y-intercept $b = 5$. The gradient is positive and it is:

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{5 - 0}{0 - (-5)} = \frac{5}{5} = 1$$

$y = mx + b$

$y = x + 5$
Example:
Find the equation of a straight line as shown in Fig.3

\[ y = 3 \]
Example:
Find the equation of a straight line as shown in Fig.4

Solution:
\[ y = -3 \]

Example: Copy and complete the table for each of these equations.

(a) \[ y = 3x + 2 \]  
(b) \[ y = 4 - \frac{3}{2}x \]  
(c) \[ 2y = -3x + 7 \]  
(d) \[ \frac{1}{3}y = 4 - \frac{3}{2}x \]
Equation | Gradient | Coordinates of y-intercept
--- | --- | ---
$y = 3x + 2$ | 3 | (0, 2)
$y = 4 - \frac{3}{2}x$ | $-\frac{3}{2}$ | (0, 4)
$2y = -3x + 7$ | $-3$ | $\left(0, \frac{7}{2}\right)$
$\frac{1}{3}y = 4 - \frac{3}{2}x$ | $-\frac{9}{2}$ | (0, 12)

**Example:** Write the equations of these **FIVE** straight lines.

(a) Line A has a gradient of 2 and y-intercept at (0, 1).

**Ans:**

\[ y = mx + b \]

\[ m = 2 \]

\[ b = 1 \]

\[ y = 2x + 1 \]

(b) Line B cuts the y-axis at -4 and has gradient of 1.

**Ans:**

\[ y = mx + b \]

\[ m = 1 \]

\[ b = -4 \]

\[ y = x - 4 \]

(c) Line C has \( m = -3 \) and \( c = \frac{1}{2} \).

**Ans:**

\[ y = mx + b \]

\[ m = -3 \]

\[ b = \frac{1}{2} \]

\[ y = -3x + \frac{1}{2} \]

(d) Line D cuts the x-axis at -4 and has gradient of 2.

**Ans:**

\[ y = mx + b \]

\[ y = 2x + b \]

\[ 0 = 2 \times (-4) + b \]

\[ b = 8 \]
\[ y = 2x + 8 \]

(f) Line E passes through the origin and has gradient of 1.

**Ans:**
\[ y = x \]

**Midpoints of coordinate pairs:**

The x-coordinate of Midpoint \( M = 2 + \frac{1}{2}(6 - 2) = 2 + \frac{4}{2} = 2 + 2 = 4 \)

The y-coordinate of Midpoint \( M = 2 + \frac{1}{2}(6 - 2) = 2 + \frac{4}{2} = 2 + 2 = 4 \)

**OR**

The x-coordinate of Midpoint \( M = \frac{x_A + x_B}{2} = \frac{6 + 2}{2} = \frac{8}{2} = 4 \)

The y-coordinate of Midpoint \( M = \frac{y_A + y_B}{2} = \frac{6 + 2}{2} = \frac{8}{2} = 4 \)
Example: Find the coordinates of the midpoint of the line segment shown in Fig. 4.

The midpoint $M$ of the line segment joining $A(x_A, y_A)$ and $B(x_B, y_B)$ is given by

$$Midpoint \; M = \frac{1}{2} \left( \frac{x_A + x_B}{2}, \frac{y_A + y_B}{2} \right) = \left( \frac{-5 + 3}{2}, \frac{-2 + 4}{2} \right) = (-1, 1)$$

Example: Find the coordinates of the midpoint of the line segment shown in Fig. 5.

The midpoint $M$ of the line segment joining $A(x_A, y_A)$ and $B(x_B, y_B)$ is given by

$$Midpoint \; M = \frac{1}{2} \left( \frac{x_A + x_B}{2}, \frac{y_A + y_B}{2} \right) = \left( \frac{-5 + 5}{2}, \frac{5 - 2}{2} \right) = (0, 1.5)$$
Example: A line segment $AB$ has midpoint $M$. Given $A$ and $M$ find the coordinates of $B$.

(a) $A(2, 2)$ and $M(3, 4)$
(b) $A(2.3, -1.3)$ and $M(-3.4, -2.5)$

(a) The midpoint $M$ of the line segment joining $A(x_A, y_A)$ and $B(x_B, y_B)$ is given by Midpoint $M$ is:

$$M = \left( \frac{x_A + x_B}{2}, \frac{y_A + y_B}{2} \right) = \left( \frac{2 + x_B}{2}, \frac{2 + y_B}{2} \right) = (3, 4)$$

$$\frac{2 + x_B}{2} = 3 \Rightarrow 2 + x_B = 6 \Rightarrow x_B = 6 - 2 = 4$$
\[ \frac{2 + y_B}{2} = 4 \Rightarrow 2 + y_B = 8 \Rightarrow y_B = 8 - 2 = 6 \]

Therefore, point B = (4, 6)

(b) The midpoint M of the line segment joining \( A(x_A, y_A) \) and \( B(x_B, y_B) \) is given by:

Midpoint M is:

\[
\left( \frac{x_A + x_B}{2}, \frac{y_A + y_B}{2} \right) = \left( \frac{2 + x_B}{2}, \frac{2 + y_B}{2} \right) = (-3.4, -2.5)
\]

\[
\frac{2.3 + x_B}{2} = -3.4 \Rightarrow 2.3 + x_B = -6.8 \Rightarrow x_B = -6.8 - 2.3 = -9.1
\]

\[
\frac{-1.3 + y_B}{2} = -2.5 \Rightarrow -1.3 + y_B = -5 \Rightarrow y_B = -5 + 1.3 = -3.7
\]

Therefore, point B = (-9.1, -3.7)

**Example:** The graphs of the line \( y = 9 - x \) and \( y = 2x + 3 \) are shown below.

(a) Use the graphs to find the solution to the simultaneous equations
\[ y = 2x + 3 \quad \text{and} \quad y = 9 - x \]

**Ans:**

Looking at the graph, the solution is: \( x = 2 \) and \( y = 7 \).

---

**Example:**

\[ y = \frac{1}{2}x \]
(a) Write down the equation of the line which is parallel to \( y = \frac{1}{2} x \) and goes through the point \((0, -2)\).

The line described by this equation crosses the x-axis at the point \( k \).

(b) Calculate the coordinate of \( k \).
(a) Use the graphs to solve the simultaneous equations

\[ 2x - y = 1 \quad 3x + 2y = 12 \]

Ans: \( x = 2 \) and \( y = 3 \) is the solution.
LESSON 11

Inequalities

$T > -3$

OR

$T > -3$

OR

$T \geq -3$

OR

$T \geq -3$

OR

$T < -2$

OR

$T < -2$

OR

$T \leq -2$

OR

$T \leq -2$
Example: Indicate the inequality, $-3 < x < 3$ on a number line, and list the whole numbers included by this inequality.

List of whole numbers: $-2, -1, 0, 1, 2$

Example: Indicate the inequality, $-6 < 3x < 12$ on a number line, and list the whole numbers included by this inequality.

Ans: 

$-6 < 3x < 12$

$\frac{6}{3} < x < \frac{12}{3}$

$-2 < x < 4$

List of whole numbers: $-1, 0, 1, 2, 3$
Example: Solve the following inequalities and show their solutions on a number line.

(a) \( 4x < 5x + 2 \)  
(b) \( 2 \leq \frac{2}{3}(x + 5) \leq 6 \)  
(c) \( -10c > 5 \)  
(d) \( 5 - 8m \leq 13 \)

Solutions

(a) \( 4x < 5x + 2 \)  
\( 4x - 5x < 2 \)  
\( -x < 2 \)  
\( x > -2 \)

(b) \( 2 \leq \frac{2}{3}(x + 5) \leq 6 \)  
\( 6 \leq 2(x + 5) \leq 18 \)  
\( \frac{6}{2} \leq (x + 5) \leq \frac{18}{2} \)  
\( 3 \leq (x + 5) \leq 9 \)  
\( 3 - 5 \leq x \leq 9 - 5 \)  
\( -2 \leq x \leq 4 \)

(c) \( -10c > 5 \)  
\( -c > \frac{5}{10} \)  
\( c < -\frac{5}{10} \Rightarrow c < -\frac{1}{2} \)

Note: When each side of the inequality is multiplied or divided by a negative number, then the sign of the inequality changes.
\( c < -0.5 \)

\( c < -\frac{1}{2} \)

(d) \( 5 - 8m \leq 13 \\
-8m \leq 13 - 5 \\
-8m \leq 8 \\
-m \leq \frac{8}{8} \\
-m \leq 1 \\
m \geq -1 \)

\( m \geq -1 \)

(a) \( \frac{x + 2}{2} > \frac{x - 1}{3} \)

Multiply both sides by the lowest common denominator (i.e. 6)
\( 3(x + 2) > 2(x - 1) \)
\( 3x + 6 > 2x - 2 \)
\( 3x - 2x > -2 - 6 \)
\( x > -8 \)

(b) \( \frac{2}{x + 2} < \frac{3}{x - 1} \)
\( 2(x - 1) < 3(x + 2) \)
\( 2x - 2 < 3x + 6 \)
\( 2x - 3x < 6 + 2 \)
\( -x < 8 \)
\( x > -8 \) \text{ Wrong! Wrong!}
\[
\frac{2(x-1) - 3(x+2)}{(x+2)(x-1)} < 0
\]
\[
\frac{2x - 2 - 3x - 6}{(x+2)(x-1)} < 0
\]
\[
-x - 8
\[
\frac{(x+2)(x-1)}{(x+2)(x-1)} < 0
\]
\[
\frac{x+8}{(x+2)(x-1)} > 0
\]

<table>
<thead>
<tr>
<th>( x = -8 )</th>
<th>( x = -2 )</th>
<th>( x = 1 )</th>
</tr>
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<tbody>
<tr>
<td>-</td>
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\(-8 < x < -2 \quad \text{OR} \quad x > 1\)

(c) \[
\frac{2-x}{2} > \frac{x-4}{3}
\]
\[
3(2-x) > 2(x-4)
\]
\[
6 - 3x > 2x - 8
\]
\[
-3x - 2x > -8 - 6
\]
\[
-5x > -14
\]
\[
x < \frac{14}{5}
\]

(d) \[
-\frac{2-x}{2} > -\frac{x-4}{3}
\]
\[
\frac{2-x}{2} > \frac{x-4}{3}
\]
\[
-2 > \frac{x-4}{3}
\]
\[
-3(2-x) > -2(x-4)
\]
\[
-6 + 3x > -2x + 8
\]
\[
3x + 2x > 8 + 6
\]
\[
5x > 14
\]
\[
x > \frac{14}{5}
\]
Inequalities Continues...

LESSON 12
\( y = -x \)

\[ y < -x \quad y > -x \]

\[ P_1(6, 6) \]
\[ P_2(7, -2) \]
\[ P_3(8, 2) \]
\[ P_4(1, -5) \]
\[ P_5(-3, -2) \]
\[ P_6(1, -1) \]
\[ P_6(1, -1) \]

\[ y = x - 6 \]
Example: Draw individual graphs to show each of the following inequalities.

(a) $x > 3$          (b) $x \geq 3$          (c) $y \geq 5$

(d) $x + y < 8$      (e) $x + y > 8$      (f) $y > 2x + 5$

(g) $y < 2x + 5$     (h) $y \geq x^2$

Ans:

(a) $x > 3$
Draw the line \( x = 3 \) and then shade the \( x > 3 \) region as follows:

**Note:** It should be noted that the required region is shaded

![Graph showing the region shaded above and to the right of the line x = 3.]

Ans: 
(b) \( x \geq 3 \)

Draw the line \( x = 3 \) and then shade the \( x \geq 3 \) region as follows:

**Note:** It should be noted that the required region is shaded

![Graph showing the region shaded above and to the right of the line x = 3.]

Ans: 
(c) \( y \geq 5 \)

Draw the line \( y = 5 \) and then shade the \( y \geq 5 \) region as follows:

**Note:** It should be noted that the required region is shaded

![Graph showing the region shaded above and to the right of the line y = 5.]

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Ans:

(d) \( x + y < 8 \)

Draw the line \( x + y = 8 \) and then shade the \( x + y < 8 \) region as follows:

Note: It should be noted that the required region is shaded

\[
\begin{array}{c|c}
 y & x \\
 8 & 0 \\
 0 & 8 \\
\end{array}
\]

\( P_1(0, 8) \) and \( P_2(8, 0) \)
Ans:
(e) $x - y < 8$

Draw the line $x - y = 8$ and then shade the $x - y < 8$ region as follows:

Note: It should be noted that the required region is shaded

$$
\begin{array}{c|c}
x - y < 8 \\
- y < -x + 8 \\
y > x - 8 \\
y = x - 8
\end{array}
$$

$P_1(0, -8) \text{ and } P_2(8, 0)$
Ans: 
(e)  \( x + y > 8 \)

Draw the line  \( x + y = 8 \) and then shade the  \( x + y > 8 \) region as follows: 

\[ \begin{array}{c|c}
  y & x \\
  \hline
  8 & 0 \\
  0 & 8 \\
\end{array} \]

\( P_1(0, 8) \) and \( P_2(8, 0) \)
Ans:

(f) \( y > 2x + 5 \)

Draw the line \( y = 2x + 5 \) and then shade the \( y > 2x + 5 \) region as follows:

**Note:** It should be noted that the required region is shaded

<table>
<thead>
<tr>
<th>( y )</th>
<th>( x )</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>-2.5</td>
</tr>
</tbody>
</table>

\( P_1(0, 5) \) and \( P_2(-2.5, 0) \)
Ans:

(g) \( y < 2x + 5 \)

Draw the line \( y = 2x + 5 \) and then shade the \( y < 2x + 5 \) region as follows:

**Note:** It should be noted that the required region is shaded

\[
\begin{array}{c|c}
\text{y} & \text{x} \\
5 & 0 \\
0 & -2.5 \\
\end{array}
\]

\( P_1(0, 5) \) and \( P_2(-2.5, 0) \)
Ans:

(h) \[ y \geq x^2 \]

Draw the line \[ y = 2x + 5 \] and then shade the \[ y \geq x^2 \] region as follows:

**Note:** It should be noted that the required region is shaded

\[
\begin{array}{c|c}
  y & x \\
  \hline
  0 & 0 \\
  1 & 1 \\
  1 & -1 \\
\end{array}
\]

\[ P_1(0, 0) \text{ and } P_2(1, 1) \text{ and } P_3(-1, 1) \]
Example: Shade and label the region represented by the inequalities \( y \leq x + 2 \), 
\( x + y < 6 \) and \( y \geq 0 \).
The graph shows the equation $x + y = 6$ and the line $y = x + 2$. The required region is the area where these two lines intersect. The points $P_1(0, 6)$ and $P_2(6, 0)$ are used to define the region. The graph also shows the line $y = 0$, which is the x-axis. The region of interest is shaded in pink. **OR**
Example: Shade and label the region represented by the inequalities \( x \geq 2, \ y \geq 1 \) and \( x + y \leq 9 \).

Example: The diagram shows the graphs of \( y = \frac{1}{2} x + 1 \), \( 5x + 6y = 30 \) and \( x = 2 \).

Shade and label with R, the region for which the point \((x, y)\) satisfies the three inequalities:

\[
\begin{align*}
y &\leq \frac{1}{2} x + 1 \\
5x + 6y &\leq 30 \\
x &\geq 2
\end{align*}
\]
Ans:
Example: The diagram shows the graphs of \( y = \frac{1}{2}x + 1 \), \( 5x + 6y = 30 \) and \( x = 2 \).

Shade and label with R, the region for which the point \((x, y)\) satisfies the three inequalities:

\[
\begin{align*}
y & \geq \frac{1}{2}x + 1 \\
5x + 6y & \leq 30 \\
x & \geq 2
\end{align*}
\]

Ans:
**Example:** Write down the three inequalities which define the triangular region ABC.

![Graph of the triangular region ABC with equations x + y = 4, x = 3, and y = 2x + 1.]

**Ans:**

\[ y \leq 2x + 1 \]
\[ x + y \geq 4 \]
\[ x \leq 3 \]

As shown as shaded region as follows:
LEESON 13

Inequalities Continues...

Example: Shade and label the region represented by the inequalities $y \leq x + 2$, $x + y \leq 6$ and $y \geq 0$.

Example: Shade and label the region represented by the inequalities $x \geq 2$, $x + y \leq 9$ and $y \geq 1$. 
Example:  The diagram shows the graphs of \( y = \frac{1}{2} x + 1 \), \( 5x + 6y = 30 \) and \( x = 2 \).

Shade and label with \( R \), the region for which the point \((x, y)\) satisfies the three inequalities: \( y \leq \frac{1}{2} x + 1 \), \( 5x + 6y \leq 30 \) and \( x \geq 2 \).

Ans:
Example: The diagram shows the graphs of \( y = \frac{1}{2} x + 1 \), \( 5x + 6y = 30 \) and \( x = 2 \). Shade and label with \( R \), the region for which the point \( (x, y) \) satisfies the three inequalities: \( y \geq \frac{1}{2} x + 1 \), \( 5x + 6y \leq 30 \) and \( x \geq 2 \).

Ans:
Example: Write down the three inequalities which define the triangular region ABC.

Ans: \( y \leq 2x + 1 \) \( x + y \geq 4 \) and \( x \leq 3 \)

As shown as shaded region as follows:
Example: Shade and label the region represented by the inequalities $x \geq 2$, $x - y \leq 9$ and $y \leq 1$.

N.B $x - y \leq 9 \Rightarrow -y \leq -x + 9 \Rightarrow y \geq x - 9$ ... Very important
LESSON 14

Quadratic Equations

The standard form of a quadratic equation is: \( ax^2 + bx + c = 0 \) and the methods for solving these equations are described as follows:

\[
\begin{align*}
ax^2 + bx + c &= 0 \\
x^2 + \frac{b}{a}x + \frac{c}{a} &= 0 \\
x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2 &= \left(\frac{b}{2a}\right)^2 + \frac{c}{a} = 0 \\
\left(x + \frac{b}{2a}\right)^2 &= \frac{b^2 - 4ac}{4a^2} = 0 \\
\left(x + \frac{b}{2a}\right)^2 &= \left(\frac{b^2 - 4ac}{4a^2}\right) \\
\left(x + \frac{b}{2a}\right) &= \pm \sqrt{\frac{b^2 - 4ac}{4a^2}} \\
x + \frac{b}{2a} &= \pm \frac{\sqrt{b^2 - 4ac}}{2a} \\
x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\
x_1 &= \frac{-b + \sqrt{b^2 - 4ac}}{2a} \quad \text{and} \quad x_2 = \frac{-b - \sqrt{b^2 - 4ac}}{2a}
\end{align*}
\]

1. Solution by Factors

In the product of two numbers is zero then one or both of them must be zero. If \( ab = 0 \) then either \( a = 0 \) or \( b = 0 \), or both \( a = 0 \) and \( b = 0 \). This fact can also be applied to the solution of quadratic equations.

Example: Solve the quadratic equation \((x - 3)(x + 1) = 0\).
Since the product of two factors is zero then one or both of them must be zero so:

If $(x - 3) = 0 \Rightarrow x = 3$

If $(x + 1) = 0 \Rightarrow x = -1$

The solution of the equation $(x - 3)(x + 1) = 0$ are $x = 3$ and $x = -1$

Example: Solve the quadratic equation $(x - 6)(2x + 1) = 0$.

Since the product of two factors is zero then one or both of them must be zero so:

If $(x - 6) = 0 \Rightarrow x = 6$

If $(2x + 1) = 0 \Rightarrow x = -\frac{1}{2}$

The solutions of the equation $(x - 6)(2x + 1) = 0$ are $x = 6$ and $x = -\frac{1}{2}$

Example: Solve the quadratic equation $(6 - x)(-2x + 1) = 0$.

Since the product of two factors is zero then one or both of them must be zero so:

If $(6 - x) = 0 \Rightarrow x = 6$

If $(-2x + 1) = 0 \Rightarrow -2x = -1 \Rightarrow x = \frac{1}{2}$

The solutions of the equation $(x - 6)(2x + 1) = 0$ are $x = 6$ and $x = \frac{1}{2}$

Example: Solve the quadratic equation $(x + a)(2x - b) = 0$.

Since the product of two factors is zero then one or both of them must be zero so:

If $(x + a) = 0 \Rightarrow x = -a$

If $(2x - b) = 0 \Rightarrow 2x = b \Rightarrow x = \frac{b}{2}$

The solutions of the equation $(x + a)(2x - b) = 0$ are $x = -a$ and $x = \frac{b}{2}$
Example: Solve the quadratic equation \( x^2 + 4x - 21 = 0 \).

\[ x^2 + 4x - 21 = (x - 7)(x + 4) = 0 \]

You need to look for two numbers which when multiplied together, gives \(-21\) and which, when added together, give \(4\). These two numbers would be: \(+7\) and \(-4\)

\[ \Rightarrow x^2 + 4x - 21 = (x + 7)(x - 4) = 0 \]

\[ \text{If } (x + 7) = 0 \Rightarrow x = -7 \]

\[ \text{If } (x - 4) = 0 \Rightarrow x = 4 \]

The solutions of the equation \( x^2 + 4x - 21 = 0 \) are \( x = -7 \) and \( x = 4 \)

Example: Solve the quadratic equation \( x^2 + 4x - 5 = 0 \).

\[ x^2 + 4x - 5 = (x - 5)(x + 1) = 0 \]

You need to look for two numbers which when multiplied together, gives \(-5\) and which, when added together, give \(4\). These two numbers would be: \(+5\) and \(-1\)

\[ \Rightarrow x^2 + 4x - 5 = (x + 5)(x - 1) = 0 \]

\[ \text{If } (x + 5) = 0 \Rightarrow x = -5 \]

\[ \text{If } (x - 1) = 0 \Rightarrow x = 1 \]

The solutions of the equation \( x^2 + 4x - 5 = 0 \) are \( x = -5 \) and \( x = 1 \)

Example: Solve the quadratic equation \( x^2 = 6x \).

\[ x^2 - 6x = 0 \]

Take \( x \) as a common factor out.

\[ x^2 - 6x = x(x - 6) = 0 \]

\[ \text{If } x = 0 \Rightarrow x = 0 \]

\[ \text{If } (x - 6) = 0 \Rightarrow x = 6 \]

The solutions of the equation \( x^2 = 6x \) are \( x = 0 \) and \( x = 6 \)

Example: Solve the quadratic equation \( x^2 - 5x + 2 = 8 \).
\[ x^2 - 5x + 2 - 8 = 0 \]
\[ x^2 - 5x - 6 = 0 \]
\[ x^2 - 5x - 6 = (x + 1)(x - 6) = 0 \]

You need to look for two numbers which when multiplied together, gives 
-6 and which, when added together, give -5. These two numbers would be: -6 and +1
\[ \Rightarrow x^2 - 5x - 6 = (x - 6)(x + 1) = 0 \]

If \((x - 6) = 0 \Rightarrow x = 6\)

If \((x + 1) = 0 \Rightarrow x = -1\)

The solutions of the equation \(x^2 - 5x + 2 = 8\) are \(x = 6\) and \(x = -1\)

**Example:** Solve the quadratic equation \(y^2 - 6y - 55 = 0\).

\[ y^2 - 6y - 55 = 0 \]
\[ y^2 - 6y - 55 = (y + 5)(y - 11) = 0 \]

You need to look for two numbers which when multiplied together, gives 
-55 and which, when added together, give -6. These two numbers would be: 
-11 and +5
\[ \Rightarrow y^2 - 6y - 55 = (y - 11)(y + 5) = 0 \]

If \((y - 11) = 0 \Rightarrow y = 11\)

If \((y + 5) = 0 \Rightarrow y = -5\)

The solutions of the equation \(y^2 - 6y - 55 = 0\) are \(y = 11\) and \(y = -5\)

**Example:** Solve the quadratic equation \(2y^2 + 5y - 3 = 0\).

**Note:** Here you need to pay a little bit more attention.

\[ 2y^2 + 5y - 3 = 0 \]
\[ 2y^2 + 5y - 3 = (2y - 1)(y + 3) = 0 \]

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If \((2y-1)=0\) \(\Rightarrow 2y=1 \Rightarrow y=\frac{1}{2}\)

If \((y+3)=0\) \(\Rightarrow y=-3\)

The solutions of the equation \(2y^2+5y-3=0\) are \(y=\frac{1}{2}\) and \(y=-3\)

**Example:** Solve the quadratic equation \(4y^2-17y-15=0\).

**Note:** Here you need to pay a little bit more attention.

\(4y^2-17y-15=0\)

\(4y^2-17y-15=(4y+3)(y-5)=0\)

If \((4y+3)=0\) \(\Rightarrow 4y=-3 \Rightarrow y=-\frac{3}{4}\)

If \((y-5)=0\) \(\Rightarrow y=5\)

The solutions of the equation \(4y^2-17y-15=0\) are \(y=-\frac{3}{4}\) and \(y=5\)

**Example:** Write down quadratic equation with solutions as \(x=3\) and \(x=4\)

**Note:** The standard equation for this is as: \((x-x_1)(x-x_2)=0\)

Here \(x_1=3\) and \(x_2=4\)

\((x-x_1)(x-x_2)=(x-3)(x-4)=0\)

\((x-3)(x-4)=0\)

\(x^2-4x-3x+12=0\)

\(x^2-7x+12=0\)

**The equation is:** \(x^2-7x+12=0\)

**Example:** Solve \(|x^2-4x-5|=7\).
Remember \( |?| = \pm (? ) \) i.e. \( |x| = \pm x \), \( |x| = \pm (x - 1) \) and so on.

\[ \pm (x^2 - 4x - 5) = 7 \]

(a) \[ x^2 - 4x - 5 = 7 \]
\[ x^2 - 4x - 5 - 7 = 0 \]
\[ x^2 - 4x - 12 = 0 \]
\[ (x - 6)(x + 2) = 0 \]

If \( x - 6 = 0 \) \( \Rightarrow x = 6 \)
If \( x + 2 = 0 \) \( \Rightarrow x = -2 \)

(b) \[ - (x^2 - 4x - 5) = 7 \]
\[ x^2 - 4x - 5 = -7 \]
\[ x^2 - 4x - 5 + 7 = 0 \]
\[ x^2 - 4x + 2 = 0 \]

\[ x_1 = \frac{-b + \sqrt{b^2 - 4ac}}{2a} \] and \[ x_2 = \frac{-b - \sqrt{b^2 - 4ac}}{2a} \]

\[ x_1 = \frac{-(-4) + \sqrt{(-4)^2 - 4(1)(2)}}{2(1)} = \frac{4 + \sqrt{16 - 8}}{2} = \frac{4 + \sqrt{8}}{2} = \frac{4 + 2 \sqrt{2}}{2} = 2 + \sqrt{2} \]
\[ x_1 = \frac{-(-4) - \sqrt{(-4)^2 - 4(1)(2)}}{2(1)} = \frac{4 - \sqrt{16 - 8}}{2} = \frac{4 - \sqrt{8}}{2} = \frac{4 - 2 \sqrt{2}}{2} = 2 - \sqrt{2} \]

The solutions are: \(-2, 6, 2 + \sqrt{2}, 2 - \sqrt{2}\)

**LESSON 15**

**Quadratic Equations Continues...**

**Example:** Solve \( x^2 + 2x - 15 = 0 \)
\[ x^2 + 2x - 15 = 0 \]
\[ (x + 5)(x - 3) = 0 \]

If \( x + 5 = 0 \) \( \Rightarrow x = -5 \)
If \( x - 3 = 0 \) \( \Rightarrow x = 3 \)

**Solving quadratic equations using quadratic formula**

**Example:** Solve \( x^2 + 2x - 15 = 0 \), using the formula: \[ x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \]

Find the value of \( x_1 \) and \( x_2 \) when Standard Form: \( ax^2 + bx + c = 0 \)

**Note:** \( a = 1, b = 2, \) and \( c = -15 \).
Example: Solve \( x^2 + 3x + 2 = 0 \) using: \( x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \)

Find the value of \( x_1 \) and \( x_2 \) when Standard Form: \( ax^2 + bx + c = 0 \)

Note: \( a = 1, b = 3, \) and \( c = 2. \)

\[
x_1 = \frac{-b + \sqrt{b^2 - 4ac}}{2a} = \frac{-3 + \sqrt{9 - 8}}{2} = \frac{-3 + 1}{2} = \frac{-2}{2} = -1
\]

\[
x_2 = \frac{-b - \sqrt{b^2 - 4ac}}{2a} = \frac{-3 - \sqrt{9 - 8}}{2} = \frac{-3 - 1}{2} = \frac{-4}{2} = -2
\]

Example: Solve \( x^2 + 3x - 10 = 0 \) using: \( x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \)

Find the value of \( x_1 \) and \( x_2 \) when Standard Form: \( ax^2 + bx + c = 0 \)

Note: \( a = 1, b = 3, \) and \( c = -10. \)

\[
x_1 = \frac{-b + \sqrt{b^2 - 4ac}}{2a} = \frac{-3 + \sqrt{9 + 49}}{2} = \frac{-3 + 7}{2} = \frac{4}{2} = 2
\]

\[
x_2 = \frac{-b - \sqrt{b^2 - 4ac}}{2a} = \frac{-3 - \sqrt{9 + 49}}{2} = \frac{-3 - 7}{2} = \frac{-10}{2} = -5
\]

Example: Solve \( 2x^2 + 5x - 3 = 0 \) using: \( x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \)

Find the value of \( x_1 \) and \( x_2 \) when Standard Form: \( ax^2 + bx + c = 0 \)

Note: \( a = 2, b = 5, \) and \( c = -3. \)

\[
x_1 = \frac{-b + \sqrt{b^2 - 4ac}}{2a} = \frac{-5 + \sqrt{25 + 42}}{4} = \frac{-5 + 7}{4} = \frac{2}{4} = 0.5
\]

\[
x_2 = \frac{-b - \sqrt{b^2 - 4ac}}{2a} = \frac{-5 - \sqrt{25 + 42}}{4} = \frac{-5 - 7}{4} = \frac{-12}{4} = -3
\]
Line of symmetry is y-axis

Line of symmetry is y-axis

Line of symmetry is y-axis

Line of symmetry is y-axis

Line of symmetry is y-axis

Line of symmetry is y-axis

Line of symmetry is x = -2

Line of symmetry is x = -2

Line of symmetry is x = -1

Line of symmetry is x = -2

Line of symmetry is x = -2

Line of symmetry is x = -1

Line of symmetry is x = -2

Line of symmetry is x = -2

Line of symmetry is x = -1
$x = -\sqrt{y + 2}$

No line of symmetry

$y = ax^2$

Line of symmetry is y-axis

$y = -x^2 + 3$

Line of symmetry is y-axis

$y = -x^2 - 1$

Line of symmetry is y-axis

$y = ax^2$

Line of symmetry is y-axis

$y = -x^2 + 1$

Line of symmetry is y-axis

$y = -x^2 - 2$

Line of symmetry is y-axis

$y = -x^2 + 2$

Line of symmetry is y-axis

$y = -x^2 - 1$

Line of symmetry is y-axis
Line of symmetry is $x$-axis

Line of symmetry is $x$-axis

Line of symmetry is $x$-axis

Line of symmetry is $x$-axis or $y = 0$

Line of symmetry is $x$-axis or $y = 0$

Line of symmetry is $x$-axis or $y = 0$

No line of symmetry

No line of symmetry

No line of symmetry
LESSON 16

Quadratic Equations Continues...

Example No: Solve \( x^2 + 2x - 15 = 0 \)
\((x + 5)(x - 3) = 0\)
If \((x + 5) = 0 \implies x = -5\)
If \((x - 3) = 0 \implies x = 3\)

Solving quadratic equations using quadratic formula

Example No: Solve \( x^2 + 2x - 15 = 0 \), using the formula: \( x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \)

Find the value of \( x_1 \) and \( x_2 \) when Standard Form: \( ax^2 + bx + c = 0 \)

Note: \( a = 1 \), \( b = 2 \), and \( c = -15 \).

\[
x_1 = \frac{-b + \sqrt{b^2 - 4ac}}{2a} = \frac{-2 + \sqrt{2^2 - 4 \times 1 \times (-15)}}{2 \times 1} = \frac{-2 + \sqrt{4 + 60}}{2} = \frac{-2 + \sqrt{64}}{2} = \frac{-2 + 8}{2} = \frac{6}{2} = 3
\]

\[
x_2 = \frac{-b - \sqrt{b^2 - 4ac}}{2a} = \frac{-2 - \sqrt{2^2 - 4 \times 1 \times (-15)}}{2 \times 1} = \frac{-2 - \sqrt{4 + 60}}{2} = \frac{-2 - \sqrt{64}}{2} = \frac{-2 - 8}{2} = \frac{-10}{2} = -5
\]

Example No: Solve \( x^2 + 3x + 2 = 0 \) using: \( x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \)

Find the value of \( x_1 \) and \( x_2 \) when Standard Form: \( ax^2 + bx + c = 0 \)

Note: \( a = 1 \), \( b = 3 \), and \( c = 2 \).

\[
x_1 = \frac{-b + \sqrt{b^2 - 4ac}}{2a} = \frac{-3 + \sqrt{3^2 - 4 \times 1 \times 2}}{2 \times 1} = \frac{-3 + \sqrt{9 - 8}}{2} = \frac{-3 + 1}{2} = \frac{-2}{2} = -1
\]

\[
x_2 = \frac{-b - \sqrt{b^2 - 4ac}}{2a} = \frac{-3 - \sqrt{3^2 - 4 \times 1 \times 2}}{2 \times 1} = \frac{-3 - \sqrt{9 - 8}}{2} = \frac{-3 - 1}{2} = \frac{-4}{2} = -2
\]

Example No: Solve \( x^2 + 3x - 10 = 0 \) using: \( x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \)

Find the value of \( x_1 \) and \( x_2 \) when Standard Form: \( ax^2 + bx + c = 0 \)

Note: \( a = 1 \), \( b = 3 \), and \( c = -10 \).
Example No: Solve \(2x^2 + 5x - 3 = 0\) using \(x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}\)

Find the value of \(x_1\) and \(x_2\) when Standard Form: \(ax^2 + bx + c = 0\)

Note: \(a = 2\), \(b = 5\), and \(c = -3\).

\[
x_1 = \frac{-b + \sqrt{b^2 - 4ac}}{2a} = \frac{-5 + \sqrt{25 + 24}}{4} = \frac{-5 + 7}{4} = \frac{2}{4} = 0.5
\]
\[
x_2 = \frac{-b - \sqrt{b^2 - 4ac}}{2a} = \frac{-5 - \sqrt{25 + 24}}{4} = \frac{-5 - 7}{4} = \frac{-12}{4} = -3
\]
\( y = (x + 2)^2 + 2 \) Line of symmetry is \( x = -2 \)

\( y = (x + 2)^2 - 2 \) Line of symmetry is \( x = -2 \)

\( y = (x + 1)^2 \) Line of symmetry is \( x = -1 \)

\( y = (x + 2)^2 - 1 \) Line of symmetry is \( x = -2 \)

\( y = (x + 1)^2 \) Line of symmetry is \( x = -1 \)

\( x = -\sqrt{y+2} \) No line of symmetry

\( x = -\sqrt{y} \) No line of symmetry

\( x = \sqrt{y} \) No line of symmetry

\( y = ax^2 \)
\( a = -3 < 0 \) Line of symmetry is y-axis

\( y = -x^2 \)
\( a = -1 < 0 \) Line of symmetry is y-axis

\( y = ax^2 \)
\( a < 0 \) Line of symmetry is y-axis
Line of symmetry is y-axis

Line of symmetry is y-axis

Line of symmetry is y-axis

Line of symmetry is y-axis

Line of symmetry is y-axis

Line of symmetry is y-axis

Line of symmetry is x-axis

Line of symmetry is x-axis

Line of symmetry is x-axis

Line of symmetry is x-axis or y = 0

Line of symmetry is x-axis or y = 0

Line of symmetry is x-axis or y = 0

Line of symmetry is x-axis or y = 0
No line of symmetry

No line of symmetry

No line of symmetry
\[ y = ax^2 \]
\[ a > 0 \]
\[ a = 3 > 0 \]

Line of symmetry is y-axis

\[ y = x^2 \]
\[ a = 1 > 0 \]
\[ (0, 0) \]

Line of symmetry is y-axis

\[ y = ax^2 \]
\[ a > 0 \]
\[ (0, 0) \]

Line of symmetry is y-axis

\[ y = x^2 + 2 \]
\[ (0, 2) \]

Line of symmetry is y-axis

\[ y = x^2 + 1 \]
\[ (0, 1) \]

Line of symmetry is y-axis

\[ y = x^2 \]
\[ (0, 0) \]

Line of symmetry is y-axis

\[ y = (x+2)^2 + 2 \]
\[ (-2, 2) \]

Line of symmetry is x = -2

\[ y = (x+2)^2 + 1 \]
\[ (-2, 1) \]

Line of symmetry is x = -2

\[ y = (x+1)^2 \]
\[ (-1, 0) \]

Line of symmetry is x = -1

\[ y = (x+2)^2 - 2 \]
\[ (-2, -2) \]

Line of symmetry is x = -2

\[ y = (x+2)^2 - 1 \]
\[ (-2, -1) \]

Line of symmetry is x = -2

\[ y = (x+1)^2 \]
\[ (-1, 0) \]

Line of symmetry is x = -1
1. $x = \sqrt{y}$
   - No line of symmetry
   - $(0, 0)$

2. $x = -\sqrt{y} + 2$
   - No line of symmetry
   - $(0, -2)$

3. $y = ax^2$
   - Line of symmetry is y-axis
   - $(0, 0)$
   - $a = -3 < 0$

4. $y = -x^2$
   - Line of symmetry is y-axis
   - $(0, 0)$
   - $a = -1 < 0$

5. $y = ax^2$
   - Line of symmetry is y-axis
   - $(0, 0)$
   - $a < 0$

6. $y = -x^2 + 3$
   - Line of symmetry is y-axis
   - $(0, 3)$

7. $y = -x^2 + 1$
   - Line of symmetry is y-axis
   - $(0, 1)$

8. $y = -x^2 + 2$
   - Line of symmetry is y-axis
   - $(0, 2)$

9. $y = -x^2 - 1$
   - Line of symmetry is y-axis
   - $(0, -1)$

10. $y = -x^2 - 2$
    - Line of symmetry is y-axis
    - $(0, -2)$

11. $y = -x^2 - 1$
    - Line of symmetry is y-axis
    - $(0, -1)$
Line of symmetry is x-axis

Line of symmetry is x-axis

Line of symmetry is x-axis

Line of symmetry is x-axis or y = 0

Line of symmetry is x-axis or y = 0

Line of symmetry is x-axis or y = 0

No line of symmetry

No line of symmetry

No line of symmetry
LEESON 17

CIRCLES

Find the coordinates of the centre and the radius of the following circles.
(a) \((x - 3)^2 + (y - 2)^2 = 36\)

**Ans:** \(C(3,2) \text{ and } r = 6\)

(b) \((x + 3)^2 + (y - 2)^2 = 16\)

**Ans:** \(C(-3,2) \text{ and } r = 4\)

(c) \(x^2 + y^2 + 3y = 10\)

**Ans:** \(x^2 + y^2 + 3y = 10\)

\[
x^2 + \left( y + \frac{3}{2} \right)^2 - \left( \frac{3}{2} \right)^2 = 10
\]

\[
x^2 + \left( y + \frac{3}{2} \right)^2 = 10 + \frac{9}{4} = \frac{49}{4}
\]

\[
x^2 + \left( y + \frac{3}{2} \right)^2 = \frac{49}{4}
\]

**Ans:** \(C\left(0, -\frac{3}{2}\right) \text{ and } r = \frac{7}{2}\)

(d) \(x^2 + 4x + y^2 - 4y - 17 = 0\)

**Ans:** \((x + 2)^2 - 4 + (y - 2)^2 - 4 - 17 = 0\)

\[(x + 2)^2 + (y - 2)^2 - 25 = 0\]

\[(x + 2)^2 + (y - 2)^2 = 25\]

**Ans:** \(C(-2,2) \text{ and } r = 5\)

(e) \(x^2 + 4x + y^2 + 6y - 12 = 0\)

**Ans:** \((x + 2)^2 - 4 + (y + 3)^2 - 9 - 12 = 0\)
\[(x + 2)^2 + (y + 3)^2 - 25 = 0\]
\[(x + 2)^2 + (y + 3)^2 = 25\]
\[C(-2, -3) \text{ and } r = 5\]

\[(f) \left(\frac{x + 2}{5}\right)^2 + \left(\frac{y + 3}{5}\right)^2 = 1\]

\[\text{Ans: } \left(\frac{x + 2}{25}\right)^2 + \left(\frac{y + 3}{25}\right)^2 = 1\]
\[(x + 2)^2 + (y + 3)^2 = 25\]
\[C(-2, -3) \text{ and } r = 5\]

**Examples**

\[(x - a)^2 + (y - b)^2 = r^2\]

The equation of a circle with a centre \((3, -1)\) and of radius 7 is:
\[(x - 3)^2 + (y - (-1))^2 = 7^2\]
\[(x - 3)^2 + (y + 1)^2 = 49\]

2. The circle represented by the equation:
\[(x + 5)^2 + (y - 1)^2 = 2\] has centre \((-5, 1)\) and radius \(\sqrt{2}\).

**Example**
The equation: \((x - 4)^2 + (y + 3)^2 = 4\), represents a circle with centre \((4, -3)\) and radius 2 units.
Multiplying out the brackets gives:

\((x^2 - 8x + 16) + (y^2 + 6y + 9) = 4\)
\(x^2 - 8x + 16 + y^2 + 6y + 9 - 4 = 0\)
\(\Rightarrow x^2 + y^2 - 8x + 6y + 16 + 9 - 4 = 0\)
\(\Rightarrow x^2 + y^2 - 8x + 6y + 21 = 0\)

**Example**
Consider the equation:
\(x^2 + y^2 + 2x - 8y + 8 = 0\)
Does this represent a circle?
Rearranging gives:
\(x^2 + y^2 + 2x - 8y = -8\)
Completing the squares in x and y gives:
\((x + 1)^2 - 1 + (y - 4)^2 - 16 = -8\)
\((x + 1)^2 - 1 + (y - 4)^2 - 16 = -8 + 17\)
\((x + 1)^2 + (y - 4)^2 = 9\)
So the equation represents a circle with centre (-1, 4) and radius 3.

**Example**
Consider the equation:
\(\frac{x^2}{9} + \frac{y^2}{9} = 1\)
\(x^2 + y^2 = 1\)

So the equation represents a circle with centre (0, 0) and radius 1.
LEESON 18

Angles

Example:  \( W \) is directly proportional to the square of \( G \), and \( W = 104 \) when \( G = 4 \)

(a) Calculate \( W \) when \( G = 6 \)

Ans:

\[
W \sim G^2 \\
W = kG^2 \\
104 = k(4)^2 \\
k = \frac{104}{16} = 6.5 \\
W = 6.5G^2 \\
W = 6.5(6)^2 = 6.5 \times 36 = 234
\]

(b) Calculate \( G \) when \( W \) is 416

Ans:

\[
W = 6.5G^2 \\
G^2 = \frac{W}{6.5}
\]
\[ G^2 = \frac{416}{6.5} \]
\[ G^2 = 64 \]
\[ G = \pm \sqrt{64} = \pm 8 \]

Example: Solve the equation
\[ x^2 + 8x - 3 = 0 \]
Giving your solutions to two decimal places.

Ans:
\[ x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \]
\[ x_{1,2} = \frac{-8 \pm \sqrt{8^2 - 4 \times 1 \times (-3)}}{2 \times 1} = \frac{-8 \pm \sqrt{64 + 12}}{2} = \frac{-8 \pm \sqrt{76}}{2} = \frac{-8 \pm 8.718}{2} \]
\[ x_1 = \frac{-8 + 8.718}{2} = \frac{0.718}{2} = 0.3589 \approx 0.36 \quad (2 \text{ dp}) \]
\[ x_2 = \frac{-8 - 8.718}{2} = \frac{-16.718}{2} = 8.359 \approx 8.36 \quad (2 \text{ dp}) \]

Example: (a) On the axes below, draw the graph of
\[ y = 2 \cos x^0 + 3 \]
(b) Use the graph to solve these equations.

(i) \[ y = 2 \cos x^0 + 3 = 4.2 \]

Ans:

The graph in part (a) represents \( y = 2 \cos x^0 + 3 \), then in the above graph, set the value of y to 4.2 and then read from the graph where y = 4.2 as shown below.
(ii) \[ 2 \cos x^0 = -0.3 \]
\[ y = 2 \cos x^0 + 3 \]
\[ 2 \cos x^0 = y - 3 = -0.3 \implies y = 3 - 0.3 = 2.7 \]
Read from the graph where \( y = 2.7 \) as shown below.
The graph shows the function $y = 2\cos x^0 + 3$ with key points:

- $(0, 5)$
- $(90^0, 3)$
- $(180^0, 1)$
- $(270^0, 3)$
- $(360^0, 5)$
Example: The diagram below shows the graph of 
\[ y = f(x) \]
(a) Draw on the graph grid below a sketch of the graph of 
\[ y = f(x) + 3 \]

(b) Draw on the graph grid below a sketch of the graph of 
\[ y = f(x+3) \]
(a) \(2 \sin 1.5x = 1.8\) for \(0^0 \leq x \leq 360^0\)

\[
\sin 1.5x = \frac{1.8}{2} = 0.9
\]

\[
1.5x = \sin^{-1}(0.9)
\]

\[
1.5x = 64.16^0, 115.84^0, 424.16^0, 475.83^0
\]

\[
x = \left[ \frac{64.16}{1.5}, \frac{115.84}{1.5}, \frac{424.16}{1.5}, \frac{475.83}{1.5} \right]
\]

\[
x = 42.77, 77.226, 282.77, 317.22
\]

(b) \(8 \cos 16x + 6 = 2\) for \(0^0 \leq x \leq 50^0\)

\[
8 \cos 16x = 2 - 6
\]

\[
8 \cos 16x = -4
\]

\[
\cos 16x = -\frac{4}{8} = -0.5
\]

\[
16x = \cos^{-1}(-0.5)
\]
16x = 120°, 240°, 480°, 600°

\[ x = \frac{120}{16} = 7.5°, \frac{240}{16} = 15°, \frac{480}{16} = 30°, \frac{600}{16} = 37.5° \]

(c) \[ 5\sin 60x + 6 = 4 \quad \text{for} \quad 0° \leq x \leq 30° \]

\[ 5\sin 60x = 4 - 6 \]
\[ \sin 60x = -2 \]
\[ \sin 60x = -\frac{2}{5} = -0.4 \]
\[ 60x = \sin^{-1}(-0.4) \]

60x = 203.58°, 336.42°, 563.58°, 696.42°, 923.58°, 1056.42°, 1283.58°, 1416.42°, 1643.58°

\[ x = 3.393°, 5.607°, 9.393°, 11.607°, 15.393°, 17.607°, 21.393°, 27.393° \]

(d) \[ 2.5\cos 3x + 12 = 10 \quad \text{for} \quad -90° \leq x \leq 90° \]

\[ 2.5\cos 3x = 10 - 12 \]
\[ 2.5\cos 3x = -2 \]
\[ \cos 3x = -\frac{2}{2.5} = -0.8 \]
\[ 3x = \cos^{-1}(-0.8) \]
\[ 3x = 143.13°, 216.87°, -143.13°, -216.87° \]
\[ x = 47.71°, 72.29°, -47.71°, -72.29° \]

(e) \[ 8\cos\left(\frac{x}{2} - 15\right) = 7 \quad \text{for} \quad 0° \leq x \leq 240° \]

\[ \cos\left(\frac{x}{2} - 15\right) = \frac{7}{8} \]
\[ \frac{x}{2} - 15 = \cos^{-1}\left(\frac{7}{8}\right) = 28.96° \]
\[ \frac{x}{2} = 28.96° + 15° = 43.96° \Rightarrow x = 87.92° \]
(f) \[-40 \cos 12x = 16 \quad \text{for} \quad 0^\circ \leq x \leq 120^\circ \]
\[
\cos 12x = \frac{-16}{40} = -0.4
\]

\[
12x = \cos^{-1}(-0.4) = 113.58^\circ, 246.42^\circ, (113.58^\circ + 360^\circ), (246.42^\circ + 360^\circ), (113.58^\circ + 720^\circ), (246.42^\circ + 720^\circ)
\]
\[
12x = (113.58^\circ + 1080^\circ), (246.42^\circ + 1080^\circ)
\]

\[
x = 9.465^\circ, 20.535^\circ, 39.465^\circ, 50.535^\circ, 69.465^\circ, 80.535^\circ, 99.465^\circ, 110.535^\circ
\]

(g) \[5 + 6 \cos(12x + 8) = 4 \quad \text{for} \quad 0^\circ \leq x \leq 90^\circ \]
\[
6 \cos(12x + 8) = 4 - 5 = -1
\]
\[
(12x + 8) = \cos^{-1}\left(-\frac{1}{6}\right) = 99.59^\circ, 260.41^\circ, 459.59^\circ, 620.41^\circ, 819.59^\circ, 980.41^\circ
\]
\[
12x = 91.59^\circ, 252.41^\circ, 451.59^\circ, 612.41^\circ, 811.59^\circ, 972.41^\circ
\]
\[
x = 7.325^\circ, 21.034^\circ, 37.633^\circ, 51.034^\circ, 67.633^\circ, 81.034^\circ
\]

**Easy**

1. \[\sec \theta = \sqrt{2} \quad \text{for} \quad 0 \leq x \leq 2\pi \]
\[
\frac{1}{\cos x} = \sqrt{2}
\]
\[
\cos x = \frac{1}{\sqrt{2}} = 0.707
\]
\[
x = \cos^{-1}(0.707) = 45^\circ = \frac{45 \times \pi}{180} = \frac{\pi}{4}
\]
\[
x = \cos^{-1}(0.707) = 315^\circ = \frac{315 \times \pi}{180} = \frac{7\pi}{4}
\]

2. \[\cosec \theta = -3 \quad \text{for} \quad 0 \leq x \leq 2\pi \]
\[
\frac{1}{\sin x} = -3
\]
\[
\sin x = -\frac{1}{3}
\]
\[
x = \sin^{-1}\left(-\frac{1}{3}\right) = 180^\circ + 19.47^\circ = 199.47^\circ = \frac{199.47 \times \pi}{180} = 3.48
\]
\[
x = \sin^{-1}\left(-\frac{1}{3}\right) = 360^\circ - 19.47^\circ = 340.53^\circ = \frac{340.53 \times \pi}{180} = 5.94
\]
\[ (3) \quad 5 \cot \theta = -2 \quad \text{for} \quad 0 \leq x \leq 2\pi \]
\[
\frac{5}{\tan \theta} = -2 \\
\tan \theta = -\frac{5}{2} = -2.5 \\
\theta = \tan^{-1}(-2.5) = 180^\circ - 68.2^\circ = 111.8^\circ = \frac{111.8 \times \pi}{180} = 1.95 \\
\theta = \tan^{-1}(-2.5) = -68.2^\circ + 360^\circ = 291.8^\circ = \frac{291.8 \times \pi}{180} = 5.09
\]

\[ (4) \quad \csc \theta = 2 \quad \text{for} \quad 0 \leq x \leq 2\pi \]
\[
\frac{1}{\sin \theta} = 2 \\
\sin \theta = \frac{1}{2} = 0.5 \\
\theta = \sin^{-1}(0.5) = 30^\circ = \frac{3 \times \pi}{180} = \frac{\pi}{3} \\
\theta = 180^\circ - 30^\circ = 150^\circ = \frac{150 \times \pi}{180} = \frac{5\pi}{6}
\]

\[ (5) \quad 3 \sec^2 \theta - 4 = 0 \quad \text{for} \quad 0^\circ \leq \theta \leq 180^\circ \]
\[
\frac{3}{\cos^2 \theta} = 4 \\
\cos^2 \theta = \frac{3}{4} = 0.75 \\
\cos \theta = \pm \sqrt{0.75} = \pm 0.866 \\
\theta = \cos^{-1}(0.866) = 30^\circ \\
\theta = \cos^{-1}(-0.866) = 180^\circ - 30^\circ = 150^\circ
\]
(6) \[ \sec(\theta - 30^\circ) = 2 \quad \text{for} \quad 0^\circ \leq \theta \leq 180^\circ \]
\[ \frac{1}{\sec(\theta - 30^\circ)} = \frac{1}{2} \]
\cos(\theta - 30^\circ) = 0.5
\theta - 30^\circ = \cos^{-1}(0.5) = 60^\circ
\theta = 60^\circ + 30^\circ = 90^\circ
\theta - 30^\circ = 360^\circ - 60^\circ = 300^\circ
\theta = 300^\circ + 30^\circ = 330^\circ \quad \text{Not acceptable} \quad \boxed{X}

(7) \[ \cot(\theta + 40^\circ) = -3 \quad \text{for} \quad 0^\circ \leq \theta \leq 180^\circ \]
\[ \frac{1}{\tan(\theta + 40^\circ)} = -3 \]
\[ \tan(\theta + 40^\circ) = -\frac{1}{3} = -0.333 \]
\theta + 40^\circ = \tan^{-1}(-0.333) = 180^\circ - 18.42^\circ = 161.58^\circ
\theta = 161.58^\circ - 40^\circ = 121.58^\circ \equiv 122^\circ
\theta = 341.58^\circ - 40^\circ = 301.58^\circ \equiv 302^\circ \quad \text{Not acceptable} \quad \boxed{X}

(8) \[ 2 \cot^2 \theta - \cot \theta = 5 \quad \text{for} \quad 0 \leq \theta \leq 2\pi \]
\[ \frac{2}{\tan^2 \theta} - \frac{1}{\tan \theta} = 5 \]
\[ \frac{2 - \tan \theta}{\tan^2 \theta} = 5 \]
\[ 5 \tan^2 \theta + \tan \theta - 2 = 0 \]
\[ \tan \theta = \frac{-1 \pm \sqrt{1^2 - 4 \times 5 \times -2}}{2 \times 5} = \frac{-1 \pm \sqrt{1 + 41}}{10} = \frac{-1 \pm \sqrt{42}}{10} = \frac{-1 \pm 6.403}{10} = 0.5403, -0.7403 \]
\theta = \tan^{-1}(-0.7403) = 180^\circ - 36.51^\circ = 143.49^\circ \equiv \frac{143.49^\circ \times \pi}{180^\circ} = 2.5
\theta = \tan^{-1}(-0.7403) = 360^\circ - 36.51^\circ = 323.49^\circ \equiv \frac{323.49^\circ \times \pi}{180^\circ} = 5.65
\theta = \tan^{-1}(0.5403) = 28.38^\circ = \frac{28.38^\circ \times \pi}{180^\circ} = 0.495
\theta = \tan^{-1}(0.5403) = 180^\circ + 28.38^\circ = 208.38^\circ = \frac{208.38^\circ \times \pi}{180^\circ} = 3.64
(8) \[2 \csc \theta - 1 = 1 \quad \text{for } 0^\circ \leq \theta \leq 180^\circ\]
\[
\frac{2}{\sin \theta} - 1 = 1
\]
\[
\frac{2}{\sin \theta} = 2
\]
\[
\sin \theta = \frac{2}{2} = 1
\]
\[
\theta = \sin^{-1}(1) = 90^\circ
\]

**Medium**

(1) \[5 \cos \theta = 3 \cot \theta \quad \text{for } 0^\circ \leq \theta \leq 360^\circ \text{ to nearest }^\circ\]
\[
5 \cos \theta = \frac{3 \cos \theta}{\sin \theta}
\]
\[
5 \sin \theta \cos \theta = 3 \cos \theta
\]
\[
5 \sin \theta \cos \theta - 3 \cos \theta = 0
\]
\[
\cos \theta(5 \sin \theta - 3) = 0
\]
If \( \cos \theta = 0 \)
\[
\theta = \cos^{-1}(0) = 90^\circ, \ 270^\circ
\]
If \( 5 \sin \theta - 3 = 0 \) \( \Rightarrow \sin \theta = \frac{3}{5} = 0.6 \)
\[
\theta = \sin^{-1}(0.6) = 37^\circ, \ 143^\circ
\]
(2) \[ \cot^2 \theta - 8 \tan \theta = 0 \quad \text{for } 0^\circ \leq \theta \leq 360^\circ \text{ to nearest } 0^\circ \]

\[ \frac{1}{\tan^2 \theta} = 8 \tan \theta \]

\[ 8 \tan^3 \theta = 1 \]

\[ \tan \theta = \frac{1}{8} \]

\[ \tan \theta = \frac{1}{2} = 0.5 \]

\[ \theta = \tan^{-1}(0.5) = 27^\circ, \ 207^\circ \]

(3) \[ 2 \sin \theta = \csc \theta \quad \text{for } 0^\circ \leq \theta \leq 360^\circ \text{ to nearest } 0^\circ \]

\[ 2 \sin \theta = \frac{1}{\sin \theta} \]

\[ \sin^2 \theta = \frac{1}{2} = 0.5 \]

\[ \sin \theta = \pm \sqrt{0.5} = \pm 0.707 \]

\[ \theta = \sin^{-1}(\pm 0.707) = 45^\circ, \ 135^\circ, \ 225^\circ, \ 315^\circ \]

(4) \[ 3 \cot \theta = \tan \theta \quad \text{for } 0^\circ \leq \theta \leq 360^\circ \text{ to nearest } 0^\circ \]

\[ \frac{3}{\tan \theta} = \tan \theta \]

\[ \tan^2 \theta = 3 \]

\[ \tan \theta = \pm \sqrt{3} = \pm 1.732 \]

\[ \theta = \tan^{-1}(\pm 1.732) = 60^\circ, \ 120^\circ, \ 240^\circ, \ 300^\circ \]
(5) \[ 4\cot \theta + 15\sec \theta = 0 \quad \text{for } 0 \leq \theta \leq 2\pi \text{ to nearest } 0^\circ \]

\[ \frac{4\cos \theta}{\sin \theta} + \frac{15}{\cos \theta} = 0 \]

\[ \frac{4\cos^2 \theta + 15\sin \theta}{\sin \theta \cos \theta} = 0 \]

\[ \sin \theta \cos \theta \neq 0 \implies \theta \neq 0^\circ, 90^\circ, 180^\circ, 270^\circ, 360^\circ \]

\[ 4\cos^2 \theta + 15\sin \theta = 0 \]

\[ 4(1 - \sin^2 \theta) + 15\sin \theta = 0 \]

\[ 4 - 4\sin^2 \theta + 15\sin \theta = 0 \]

\[ 4\sin^2 \theta - 15\sin \theta - 4 = 0 \]

\[ (4\sin \theta + 1)(\sin \theta - 4) = 0 \]

If \( 4\sin \theta + 1 = 0 \) \implies \sin \theta = -\frac{1}{4} = -0.25

\[ \theta = \sin^{-1}(-0.25) = 165.52^\circ = \frac{165.52 \times \pi}{180} = 2.89, \quad 345.52^\circ = \frac{345.52 \times \pi}{180} = 6.03 \]

If \( \sin \theta - 4 = 0 \) \implies \sin \theta = 4 \quad \text{Not acceptable} \quad \times

(6) \[ \sec \theta = 2\cos \theta \quad \text{for } 0 \leq \theta \leq 2\pi \text{ to nearest } 0^\circ \]

\[ \frac{1}{\cos \theta} = 2\cos \theta \]

\[ 2\cos^2 \theta = 1 \]

\[ \cos^2 \theta = \frac{1}{2} = 0.5 \]

\[ \cos \theta = \pm \sqrt{0.5} = \pm 0.707 \]

\[ \theta = \cos^{-1}(\pm 0.707) = 45^\circ = \frac{\pi}{4}, \quad 135^\circ = \frac{3\pi}{4}, \quad 225^\circ = \frac{5\pi}{4}, \quad 315^\circ = \frac{7\pi}{4} \]
\( 3 \cot \theta = 2 \sin \theta \quad \text{for} \quad 0 \leq \theta \leq 2\pi \) to nearest \( ^\circ \)

\[
\frac{3 \cos \theta}{\sin \theta} = 2 \sin \theta \\
2 \sin^2 \theta = 3 \cos \theta \\
2(1 - \cos^2 \theta) - 3 \cos \theta = 0 \\
2 - 2\cos^2 \theta - 3 \cos \theta = 0 \\
-2 \cos^2 \theta - 3 \cos \theta + 2 = 0 \\
2 \cos^2 \theta + 3 \cos \theta - 2 = 0 \\
(2 \cos \theta - 1)(\cos \theta + 2) = 0 \\
\]

\( \text{If} \quad 2 \cos \theta - 1 = 0 \quad \Rightarrow \cos \theta = \frac{1}{2} = -0.5 \)

\[ \theta = \cos^{-1}(-0.5) = 120^\circ = \frac{120 \times \pi}{180} = \frac{2 \pi}{3} \]

\[ 240^\circ = \frac{240 \times \pi}{180} = \frac{4 \pi}{3} \]

\( \text{If} \quad \cos \theta + 2 = 0 \quad \Rightarrow \cos \theta = -2 \quad \text{Not acceptable} \)

\( 2 \cot^2 \theta - \cot \theta = 5 \quad \text{for} \quad 0 \leq \theta \leq 2\pi \)

\[
\frac{2}{\tan^2 \theta} - \frac{1}{\tan \theta} = 5 \\
2 - \tan \theta = 5 \\
5 \tan^2 \theta + \tan \theta - 2 = 0 \\
\]

\[
\tan \theta = \frac{-1 \pm \sqrt{1^2 - 4 \times 5 \times -2}}{2 \times 5} = \frac{-1 \pm \sqrt{1 + 41}}{10} = \frac{-1 \pm 6.403}{10} = \frac{-1 \pm 6.403}{10} = 0.5403, -0.7403 \\
\]

\( \theta = \tan^{-1}(0.5403) = 180^\circ + 36.51^\circ = 143.49^\circ = \frac{143.49 \times \pi}{180} = 2.5 \)

\( \theta = \tan^{-1}(-0.7403) = 360^\circ - 36.51^\circ = 323.49^\circ = \frac{323.49 \times \pi}{180} = 5.65 \)

\( \theta = \tan^{-1}(0.5403) = 28.38^\circ = \frac{28.38 \times \pi}{180} = 0.495 \)

\( \theta = \tan^{-1}(0.5403) = 180^\circ + 28.38^\circ = 208.38^\circ = \frac{208.38 \times \pi}{180} = 3.64 \)
Medium to Hard

(1) \[ \cos \theta + \sin \theta \tan \theta = \sec \theta \quad \text{for} \quad 0^\circ \leq \theta \leq 180^\circ \quad \text{to nearest} \quad 0^\circ \]

\[ \cos \theta + \sin \theta \left( \frac{\sin \theta}{\cos \theta} \right) = \frac{\cos^2 \theta + \sin^2 \theta}{\cos \theta} = \frac{1}{\cos \theta} = \sec \theta \quad \Rightarrow \quad \text{ok} \]

\[ \sec \theta = -3 \]

\[ \cos \theta = -\frac{1}{3} \]

\[ \theta = \cos^{-1} \left( -\frac{1}{3} \right) = 180^\circ - 70.55^\circ = 109.45^\circ \]

\[ \theta = \cos^{-1} \left( -\frac{1}{3} \right) = 360^\circ - 70.55^\circ = 289.45^\circ \quad \times \]

(2) \[ 4 \cos \theta - 3 \sec \theta = 2 \tan \theta \quad \text{for} \quad 0 \leq \theta \leq \pi \]

\[ 4 \cos \theta - \frac{3}{\cos \theta} = 2 \frac{\sin \theta}{\cos \theta} \]

\[ \frac{4 \cos^2 \theta - 3}{\cos \theta} = 2 \frac{\sin \theta}{\cos \theta} \]

\[ 4 \cos^2 \theta - 3 = 2 \sin \theta \]

\[ 4(1 - \sin^2 \theta) - 3 - 2 \sin \theta = 0 \]

\[ 4 - 4 \sin^2 \theta - 3 - 2 \sin \theta = 0 \]

\[ 4 \sin^2 \theta + 2 \sin \theta - 1 = 0 \]

\[ \sin \theta = \frac{-2 \pm \sqrt{2^2 - 4 \times 4 \times -1}}{2 \times 4} = \frac{-2 \pm \sqrt{4 + 16}}{8} = \frac{-2 \pm \sqrt{20}}{8} = \frac{-2 \pm 4.472}{8} = 0.309, -0.809 \]

\[ \theta = \sin^{-1} (0.309) = 17.998^\circ \approx 18^\circ \]

\[ \theta = \sin^{-1} (-0.809) = 180^\circ + 54^\circ \approx 234^\circ \quad \times \]

\[ \theta = \sin^{-1} (-0.809) = 360^\circ - 54^\circ \approx 306^\circ \quad \times \]
(3) Show that \[
\frac{\sin \theta}{1 + \cos \theta} + \frac{1 + \cos \theta}{\sin \theta} = 2 \csc \theta
\]

and hence solve \[
\frac{\sin \theta}{1 + \cos \theta} + \frac{1 + \cos \theta}{\sin \theta} = \frac{\sin \theta}{\sin \theta(1 + \cos \theta)} + \frac{\sin \theta}{\sin \theta(1 + \cos \theta)} = \frac{\sin^2 \theta + 1 + \cos \theta + \cos \theta + \cos^2 \theta}{\sin \theta(1 + \cos \theta)} = \frac{\sin^2 \theta + 1 + \cos \theta + \cos \theta + \cos^2 \theta}{\sin \theta(1 + \cos \theta)}
\]

\[
= \frac{1 + 1 + 2 \cos \theta}{\sin \theta(1 + \cos \theta)} = \frac{2(1 + \cos \theta)}{\sin \theta(1 + \cos \theta)} = \frac{2}{\sin \theta} = 2 \sec \theta
\]

\[
2 \sec \theta = \frac{2}{\sin \theta} = -\frac{4}{\sqrt{3}} \Rightarrow \sin \theta = -\frac{2\sqrt{3}}{4} = -\frac{\sqrt{3}}{2}
\]

\[
\theta = \sin^{-1} \left(-\frac{\sqrt{3}}{2}\right) = 180 + 60^\circ = 240^\circ
\]

\[
\theta = \sin^{-1} \left(-\frac{\sqrt{3}}{2}\right) = 360 - 60^\circ = 300^\circ
\]

(4) Show that \[
\frac{\cos \theta}{1 - \sin \theta} + \frac{1 - \sin \theta}{\cos \theta} = 2 \sec \theta
\]

and hence solve \[
\frac{\cos \theta}{1 - \sin \theta} + \frac{1 - \sin \theta}{\cos \theta} = \frac{\cos \theta}{\cos \theta(1 - \sin \theta)} + \frac{1 - \sin \theta}{\cos \theta(1 - \sin \theta)} = \frac{\cos \theta + (1 - \sin \theta)(1 - \sin \theta)}{\cos \theta(1 - \sin \theta)} = \frac{\cos \theta + 1 - \sin \theta - \sin \theta + \sin^2 \theta}{\cos \theta(1 - \sin \theta)}
\]

\[
= \frac{1 + 1 - 2 \sin \theta}{\cos \theta(1 - \sin \theta)} = \frac{2(1 - \sin \theta)}{\cos \theta(1 - \sin \theta)} = \frac{2}{\cos \theta} = 2 \sec \theta \quad \text{ok}
\]

\[
2 \sec \theta = \frac{2}{\cos \theta} = -2 \Rightarrow \cos \theta = \frac{-2}{2} = -1
\]

\[
\theta = \cos^{-1}(-1) = 180^\circ = \frac{180\pi}{180} = \pi
\]
Angle inscribed in a semicircle

Thales’ Theorem

Thales' Theorem states that the diameter of a circle subtends a right angle to any point of the circle's circumference. No matter where the point is, the triangle formed is always a right triangle.

The angle inscribed in a semicircle is always 90°.

1. Find the angles of triangle ABC if AC is the diameter of the circle:

\[ \angle ABC = 90° \]
\[ \angle ACB = 180° - (90° + 35°) = 55° \]
2. PQ is the diameter of the circle and RP = RQ. Find all the angles of the triangle.

Ans

\[ \angle PRQ = 90^\circ \]
\[ \angle RPQ = \angle RQP = \frac{180 - 90}{2} = \frac{90}{2} = 45^\circ \]

3. Find angles CBA and ABD.

Ans

\[ \angle CBA = 180 - (90 + 25) = 65^\circ \]
\[ \angle ABD = 180 - (90^\circ + 40^\circ) = 50^\circ \]

4. Find the angles POR and POQ.

Ans

\[ \angle POR = \angle POQ = 42^\circ \]
\[ \angle POR = 180 - (90^\circ + 42^\circ) = 48^\circ \]
\[ \angle POQ = \angle POR + \angle QOR = 48 + 48 = 96^\circ \]

5. Find the angle XYZ.

Ans

\[ \angle OTX = \angle OTZ = 90^\circ \]
\[ \angle OXT = \angle OZT = 90^\circ - 56^\circ = 34^\circ \]
\[ \angle OZY = \angle OYZ = 90^\circ - 34^\circ = 56^\circ = \angle XYZ \]
\[ \Rightarrow \angle XYZ = 56^\circ \]

6. The angle formed by a radius and a tangent to the circle is a right angle

\[ \angle OAT = 90^\circ \]
7. Tangents to a circle from an external point are equal: $PT = PS$

8. The angle between the tangent to a circle and a chord is equal to the angle in the alternate, or opposite, segment.

$$\angle QPT = \angle PRQ$$

9. The angle at the centre is twice the angle at the circumference subtended by the same arc.

$$\angle AOB = 2\angle ACB$$
10. Angles in the same segment of a circle are equal (i.e. angles subtended at the circumference by the same arc).
\[ \angle ACB = \angle ADB \]
\[ \angle CAD = \angle CBD \]

11. The opposite angles of a cyclic quadrilateral are supplementary. (A cyclic quadrilateral is any quadrilateral whose vertices are points on the circumference of a circle).
\[ \angle ABC + \angle ADC = 180^\circ \] and \[ \angle BAD + \angle BCD = 180^\circ \]

Note: \( n \) represents the number of sides and \( T \) represents the numbers of triangles

Note: Sum of interior angles of any polygon = \((n - 2)\times 180\)

Note: Each interior angle of any polygon = \(\frac{(n - 2)\times 180}{n}\)

Note: Exterior Angle + Interior = 180°
Exercise

1. Find $\angle PST$

![Diagram of PST with angles]

Ans: PST is an isosceles triangle (i.e. PS = PT), then $\angle PST = \angle PTS$

$\angle PST = \frac{180 - 70}{2} = \frac{110}{2} = 55^\circ$

2. Find $\angle SPT$

![Diagram of SPT with angles]

Ans:

$\angle OSP = 90^\circ$ and $\angle OPS = 180 - (55 + 90) = 35^\circ$ and $\angle OPT = \angle OPS = 35^\circ$

$\angle SPT = \angle OSP + \angle OPT = 35^\circ + 35^\circ = 70^\circ$

(3a) Find $\angle OPQ$

Ans:

OPQ triangle is an isosceles and OP = OQ

and $\angle OPQ = \angle OQP = \frac{180 - 74}{2} = 53^\circ$

(3b) Find $\angle BCD$

Ans:

$\angle BAD + \angle BCD = 180^\circ$

$62^\circ + \angle BCD = 180^\circ$

$\Rightarrow \angle BCD = 180^\circ - 62^\circ = 118^\circ$
4. Find the angles of the triangle DEB

\[ \angle CEA = \angle DEB = 180 \quad (52 + 28) = 180 \quad 80 = 100^\circ \]

\[ \angle ECA = \angle EDB = 52^\circ \quad \text{and} \quad \angle EAC = \angle EBD = 28^\circ \]

5. Find \( \angle BCA \) and \( \angle CAD \)

\[ \angle BCA = 180 \quad (19 + 90) = 71^\circ \]

\[ \angle BAD + \angle BCD = 180^\circ \]

\[ 19^\circ + \angle CAD + (71 + 42) = 180^\circ \]

\[ \angle CAD = 180^\circ - 19 - 113 = 48^\circ \]

6. Find \( \angle QSR \) and \( \angle QRS \) and \( \angle SQR \)

\[ \angle QSR = 32^\circ \]

\[ \angle QPS + \angle QRS = 180^\circ \]

\[ 74^\circ + \angle QRS = 180^\circ \]

\[ \angle QRS = 180^\circ - 74 = 106^\circ \]

\[ \angle SQR = 180^\circ - 106 - 32 = 42^\circ \]

7. AB and BC are equal in length

Find \( \angle ACB \) and \( \angle ADB \)

\[ \angle ACB = 41^\circ \]

\[ \angle ADB = \angle CAB = 41^\circ \]

8. Angle PTQ = 25°, RT is a diameter, and TS=SR. Find angles: QTR, TRQ and SRQ
\[ \angle QTR = 90^0 - 25^0 = 65^0 \quad \angle TRQ = 180^0 - (90^0 + 65^0) = 25^0 \]
\[ \angle SRT = \frac{180^0 - 90^0}{2} = 45^0 \quad \text{and} \quad \angle SRQ = \angle SRT + \angle TRQ = 45^0 + 25^0 = 70^0 \]

9. Find \( \angle APB \) \( \angle AOB \) \( \angle ACB \)

\[ \angle APB = 180^0 - (65^0 + 65^0) = 180^0 - 130^0 = 50^0 \]
\[ \angle OAB = \angle OBA = 90^0 - 65^0 = 25^0 \]
\[ \angle AOB = 180^0 - (25^0 + 25^0) = 180^0 - 50^0 = 130^0 \]
\[ \angle ACB = \frac{\angle AOB}{2} = \frac{130^0}{2} = 65^0 \]

10. O is the centre of the circle. \( \angle AOB = 108^0 \), \( \angle ACO = 33^0 \). Find angle \( \angle OBC \)

\[ \angle ACB = \frac{108^0}{2} = 54^0 \]
\[ \angle OCB = 54^0 - 33^0 = 21^0 \]
\[ \angle OBC = \angle OCB = 21^0, \text{ because } OC = OB = \text{radius} \]

11. AB is a diameter of the circle. Find: \( \angle DCB \), \( \angle DAB \) and \( \angle AXD \)
12 BDEF is a cyclic quadrilateral. EFA is a straight line. CBA is a tangent to the circle. Angle $\angle EDB = 63^0$ and angle $\angle FBA = 38^0$.

(a) Calculate the size of the angle BAF showing clearly how you found your answer.

(b) Calculate the size of the angle EBF showing clearly how you found your answer.

$\angle BEF = 38^0$

$\angle EFB = 180^0 - 63^0 = 117^0$

$\angle BFA = 180^0 - 117^0 = 63^0$

$\angle BAF = 180^0 - (38^0 + 63^0) = 180^0 - 101^0 = 79^0$

$\angle EBF = 180^0 - (\angle BEF + \angle BAF + 38^0) = 180^0 - (38^0 + 79^0 + 38^0) = 180^0 - 155^0 = 25^0$

$\angle a + \angle c = 180^0$

$\angle b + \angle d = 180^0$
LESSON 19

PROBABILITIES

Example: There has been a problem on a car production line, and all the cars need to be recalled for a check. The probability that one of these cars has faulty steering is 0.1 and the probability that it has a faulty handbrake is 0.2. These probabilities are independent of each other.

(a) Fill in the probabilities on the tree diagram below.

(b) Using the answer to (a), or otherwise, find the probability that one of these cars has:

(i) both fault.

Ans:

\[ 0.1 \times 0.2 = 0.02 \]
(ii) neither fault.
Ans:
\[0.9 \times 0.8 = 0.72\]

(iii) only one fault.
Ans:
\[(0.1 \times 0.8) + (0.9 \times 0.2) = 0.08 + 0.18 = 0.26\]

Example: Albert does an experiment with a buttered scone. He drops it 10 times and finds that it lands with its butter side down 7 times.

(a) What is the experimental probability of the scone landing butter side down?
Ans:
\[
\frac{7}{10}
\]

(b) Based on this probability, how many times would you expect the carpet to get buttered if the scone was dropped 50 times?
Ans:
\[
\frac{7}{10} \times 50 = 35
\]

Example: A postbag contains letters from abroad. There are 4 letters from Italy, 10 from France and 6 from Belgium. Two letters are removed at random without replacement. Calculate the probability that they come from the same country.
 Ans:
\[
\text{Total number of letters} = 4 + 10 + 6 = 20
\]
\[
P(\text{Both from Italy}) = \frac{4}{20} \times \frac{3}{19} = \frac{12}{380}
\]
\[
P(\text{Both from France}) = \frac{10}{20} \times \frac{9}{19} = \frac{90}{380}
\]
\[
P(\text{Both from Belgium}) = \frac{6}{20} \times \frac{5}{19} = \frac{30}{380}
\]
\[
P(\text{Both from same country}) = \frac{12}{380} + \frac{90}{380} + \frac{30}{380} = \frac{132}{380} = 0.347
\]
Example: Two boxes contain coloured bricks.

Box A contains 2 red bricks, 3 blue bricks and 1 yellow brick.

Box B contains 3 red bricks, 2 yellow bricks and 1 green brick.

Janet selects one brick from box A and one brick from box B.

Calculate the probability that the two bricks will be of the same colour.

Ans:

$$P = \frac{2}{6} \times \frac{3}{6} + \frac{3}{6} \times \frac{0}{6} + \frac{1}{6} \times \frac{2}{6} + \frac{0}{6} \times \frac{1}{6} + \frac{6}{36} + \frac{0}{36} = \frac{8}{36} = \frac{2}{9}$$

Example: Ahmad, Omar and Obaid are shooting at a target.

The probability of Ahmad hitting the target is 0.5
The probability of Omar missing the target is 0.4
The probability of Obaid hitting the target is 0.2

If each of them fires one shot at the target, find the probability that:

(a) All three hit the target.
Ans: \(0.5 \times 0.6 \times 0.2 = 0.06\)

(b) None of them hit the target.
Ans: \((1 - 0.5) \times 0.4 \times (1 - 0.2) = 0.5 \times 0.4 \times 0.8 = 0.16\)
(c) At least one of them hit the target.

\[ \text{Ans: } 1 - 0.16 = 0.84 \]

OR

Key: h = hit and m = miss

\[
\begin{array}{cccc}
1 & 1 & 1 & h \\
1 & 1 & 0 & h \\
1 & 0 & 1 & h \\
1 & 0 & 0 & m \\
0 & 1 & 1 & m \\
0 & 1 & 0 & m \\
0 & 0 & 1 & m \\
0 & 0 & 0 & m \\
\end{array}
\]

\[
\begin{array}{cc}
\text{Ahmad} & \text{Omar} \\
0.5 & 0.6 \\
0.5 & 0.4 \\
0.5 & 0.2 \\
0.5 & 0.8 \\
\end{array}
\]

\[ \text{OR } \text{At least one of them hit the target.} \]

\[ \text{Ans: } (0.5 \times 0.6 \times 0.2) + (0.5 \times 0.6 \times 0.8) + (0.5 \times 0.4 \times 0.2) + (0.5 \times 0.4 \times 0.8) +
(0.5 \times 0.6 \times 0.2) + (0.5 \times 0.6 \times 0.8) +
(0.5 \times 0.4 \times 0.2) = 0.06 + 0.24 + 0.04 + 0.16 + 0.06 + 0.24 + 0.04 = 0.84 \]

Example: Bridget is given 11 coloured sweets:
3 of them green, 1 of them is black, and 7 of them are red.

(a) (i) She eats one of the sweets. Assuming that she is equally fond of each sort of sweet, what is the probability that the sweet she eats is red?

\[ \text{Ans: } \frac{\text{Red}}{\text{Total}} = \frac{7}{3 + 1 + 7} = \frac{7}{11} \]

(ii) Bridget then eats a second sweet. What is the probability that the first sweet is red and the second is black?

\[ \text{Ans: } \frac{7}{11} \times \frac{1}{10} = \frac{7}{110} \]
(b) What is the probability that the first two sweets Bridget eats are both the same colour?

**Ans:**

\[
\left( \frac{7}{11} \times \frac{6}{10} \right) + \left( \frac{3}{11} \times \frac{2}{10} \right) = \frac{24}{55}
\]

**Example:** Ahmad, Omar and Obaid are shooting at a target.
The probability of Ahmad hitting the target is 0.5
The probability of Omar hitting the target is 0.4
The probability of Obaid hitting the target is 0.2

If each of them fires one shot at the target, find the probability that:

(d) All three hit the target.

**Ans:** \(0.5 \times 0.4 \times 0.2 = 0.04\)

(e) None of them hit the target.

**Ans:** \((1 - 0.5) \times (1 - 0.4) \times (1 - 0.2) = 0.5 \times 0.6 \times 0.8 = 0.24\)

(f) At least one of them hit the target.

**Ans:** \(1 - 0.24 = 0.76\)

**OR**

**Ans:**

\[
0.2 \times (1 - 0.5) \times (1 - 0.4) + 0.4 \times (1 - 0.5) \times (1 - 0.2) + 0.2 \times 0.4 \times (1 - 0.5) + 0.5 \times (1 - 0.2) \times (1 - 0.4) \\
+ 0.2 \times 0.5 \times (1 - 0.4) + 0.5 \times 0.4 \times (1 - 0.2) + 0.2 \times 0.4 \times 0.5
\]

**Ans:**

\[
(0.2 \times 0.5 \times 0.6) + (0.4 \times 0.5 \times 0.8) + (0.2 \times 0.4 \times 0.5) + (0.5 \times 0.8 \times 0.6) \\
+ (0.2 \times 0.5 \times 0.6) + (0.5 \times 0.4 \times 0.8) + (0.2 \times 0.4 \times 0.5)
\]

\[
= 0.06 + 0.16 + 0.04 + 0.24 + 0.06 + 0.16 + 0.04
\]

\[
= 0.76
\]
LESSON 20

ADDITIONAL QUESTIONS

1. £850 is invested for 2 years at 6% per annum compound interest.
   
   (a) Work out the total interest earned over the first 2 years.

   Ans: Formula: \( A = P(1 + r)^n \) where

   \( P \) is the principal (the initial amount you borrow or deposit)
   \( r \) is the annual rate of interest (percentage)
   \( n \) is the number of years the amount is deposited or borrowed for.
   \( A \) is the amount of money accumulated after \( n \) years, including interest.

   When the interest is compounded once a year:

   \[ A = P(1 + r)^n = 850(1 + 0.06)^2 = £955.06 \Rightarrow \text{Interest} = £955.06 - £850 = £105.06 \]

   (b) How much money will be in the account after 3 years?

   Ans: \( A = P(1 + r)^n = 850(1 + 0.06)^3 = £1012.36 \)

2. (a) Circle the numbers below which are irrational.

   \( \frac{\pi}{6} \quad \frac{3}{4} \quad 0.47\dot{1} \quad 8 \quad \sqrt{2} \quad 0.5832 \)

   Ans: \( \frac{\pi}{6} \quad \frac{3}{4} \quad 0.47\dot{1} \quad 8 \quad \sqrt{2} \quad 0.5832 \)

   (b) (i) Write \( \frac{3}{20} \) as a decimal.

   Ans:

   \[ \frac{3}{20} = \frac{15}{100} = 0.15 \]

   (i) Write \( \frac{5}{6} \) as a recurring decimal.

   Ans:

   \[ \frac{5}{6} = 0.8\dot{3} \]
(c )  **Express**  \(0.\overline{54}\)  as a fraction in its lowest terms.
**Ans:**

Let \(x = 0.\overline{54}\)

\(i.e.\  x = 0.\overline{54} \overline{54} ...\)

Then \(100x = 54.\overline{54}\)

\(So\ 99x = 54.\overline{54} - 0.\overline{54} = 54\)

\[x = \frac{54}{99} = \frac{6}{11}\]

3. Abdul Rahim measured the heights of the pupils in his class. He calculated the following results from his data:

<table>
<thead>
<tr>
<th>Girls</th>
<th>Boys</th>
</tr>
</thead>
<tbody>
<tr>
<td>Smallest height = 136 cm</td>
<td>Range = 38 cm</td>
</tr>
<tr>
<td>Largest height = 179 cm</td>
<td>Smallest height = 132 cm</td>
</tr>
<tr>
<td>Lower quartile = 151 cm</td>
<td>Inter-quartile range = 9 cm</td>
</tr>
<tr>
<td>Upper quartile = 162 cm</td>
<td>Upper quartile = 161 cm</td>
</tr>
<tr>
<td>Median = 157 cm</td>
<td>Median = 155 cm</td>
</tr>
</tbody>
</table>

On the grid below, draw 2 box-plots, one for boys and one for girls.
4. (a) $x = 0.4\text{ to }1\text{ significant figure.}$
  $y = 20\text{ to }1\text{ significant figure.}$

  Calculate the least possible value of $y - 7 - x$.

  Ans: $x = 0.35 \pm 0.4\text{ (1.s.f.) and } y = 15 \pm 20\text{ (1.s.f.)}$
  The least possible value of $y - 7 - x = 15 - 7 - 0.35 = 7.65$.

5. Mr. Ahmad and Mr. Omar are dentists. Each dentist asks 20 of his own patients to give him a customer satisfaction rating between 1 and 10 using the scale below.

<table>
<thead>
<tr>
<th>Extremely satisfied</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>Completely satisfied</th>
</tr>
</thead>
<tbody>
<tr>
<td>Increasingly dissatisfied</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>5</td>
<td>6</td>
<td>7</td>
<td>8</td>
<td>9</td>
<td>Increasingly satisfied</td>
</tr>
</tbody>
</table>

Mr. Ahmad’s results are as follows:

10, 10, 9, 3, 7, 6, 4, 9, 9, 5, 2, 6, 6, 4, 7, 8, 9, 7, 3, 1.

Mr. Omar works out that the mean of his result is 7.2, the median is 7, the model is 6 and the rage is 4.

(a) Calculate the mean, mode and median and range of Mr. Ahmad’s results.

Ans:

$mean = \frac{(10 + 10 + 9 + 3 + 7 + 6 + 4 + 9 + 9 + 5 + 2 + 6 + 6 + 4 + 7 + 8 + 9 + 7 + 3 + 1)}{20}$
Note: Put the data in correct order for median:
1, 2, 3, 3, 4, 4, 5, 6, 6, 6, 7, 7, 8, 9, 9, 9, 9, 10, 10.

Median = \frac{1}{2} (n + 1)^{th} term = \frac{1}{2} (20 + 1) = 11.5^{th} term = \frac{1}{2} (6 + 7) = \frac{13}{2} = 6.5

Mode = 9
The range = 10 - 1 = 9

Mr. Ahmad makes the following statement:
“The customers at my surgery are more satisfied than those at Mr. Omar’s.”

(b) Give one reason why Mr. Ahmad may be considered correct.
Ans:
Mr. Ahmad might be considered correct because the mode for his data is 9, which is higher than Mr. Omar’s mode.

(c) Give one reason why Mr. Ahmad may be considered wrong.
Ans:
Mr. Ahmad might be considered wrong because his median is 6.5, which is lower than Mr. Omar’s. OR Mr. Ahmad might be considered wrong because his mean is 6.25, which is lower than Mr. Omar’s. OR Mr. Ahmad might be considered wrong because Mr. Omar’s does not have any extremely dissatisfied customers (his range is only 4), whereas Mr. Ahmad does.

6. Abdul Rahman believes that the hour of sunshine per month in Kabul is inversely proportional to the depth of rainfall recorded in one month in Kabul. In one month a total of 14 cm of rain was recorded whilst there were 145 hours of sunshine.

How much rain would Abdul Rahman expect if Kabul got 90 hours of sunshine in a month?
Ans:
\[ h \propto \frac{1}{r} \]
\[ h = \frac{k}{r} \]
\[ 145 = \frac{k}{14} \]
\[ k = 145 \times 14 = 2030 \text{ hr cm} \]
\[ h = \frac{2030}{r} \]
7. A net for a cone is shown below.

![Net of a cone](image)

(a) Calculate the area of the net. Give the units of your answer.
\[
\text{Area} = \frac{280}{360} \times \pi \times r^2 = \frac{280}{360} \times \pi \times (21)^2 = 1077.56 = 1080 \text{ cm}^2 (3 \text{ s.f})
\]

(b) Calculate the perimeter of the net.
\[
\text{Arc length} = \frac{280}{360} \times 2\pi r = \frac{280}{360} \times 2 \times \pi \times 21 = 102.62 \text{ cm}
\]
\[
\text{Perimeter} = 102.62 + 21 + 21 = 144.62 = 145 \text{ cm (3 s.f)}
\]

8. Abid does an experiment with a buttered scone. He drops it 10 times and finds that it lands with its butter side down 7 times.

(a) What is the experimental probability of the scone landing butter side down?
\[
\text{Ans:} \quad \frac{7}{10}
\]

(b) Based on this probability, how many times would you expect the carpet to get buttered if the scone was dropped 50 times?
\[
\text{Ans:} \quad \frac{7}{10} \times 50 = 35
\]
9. A postbag contains letters from abroad. There are 4 letters from Kabul, 10 from Herat and 6 from Ghazni. Two letters are removed at random without replacement. Calculate the probability that they come from the same country.

Ans:

Total number of letters = 4 + 10 + 6 = 20

\[ P(Both \ from \ Kabul) = \frac{4 \times \frac{3}{19}}{20} = \frac{12}{380} \]

\[ P(Both \ from \ Herat) = \frac{10 \times \frac{9}{19}}{20} = \frac{90}{380} \]

\[ P(Both \ from \ Ghazni) = \frac{6 \times \frac{5}{19}}{20} = \frac{30}{380} \]

\[ P(Both \ from \ same \ City) = \frac{12}{380} + \frac{90}{380} + \frac{30}{380} = \frac{132}{380} = 0.347 \]

10. 2 7 12 17 22 ....... .......

(a) What are the next 2 terms of this sequence?

Ans: The next 2 terms are: 27 and 32.

(b) Write a formula for the \( n^{th} \) term of the sequence.

Ans: It is Arithmetic series with \( d = 5 \)

\[ n^{th} \ term = l = a + (n - 1)d = 2 + 5(n - 1) = 5n - 3 \]

(c) What is the 100\(^{th}\) term of this sequence?

Ans:

\[ n^{th} \ term = 5n - 3 \]

100\(^{th}\) term = 5 \times 100 - 3 = 497

OR

100\(^{th}\) term = \( a + (n - 1)d = 2 + 5(100 - 1) = 2 + 5 \times 99 = 2 + 495 = 497 \)

11. (a) Calculate \( \frac{3}{4} \div \frac{2}{3} \). Give your answer as a mixed number.

Ans:

\[ \frac{3}{4} \div \frac{2}{3} = \frac{11}{4} \div \frac{2}{3} = \frac{11 \times 3}{4 \times 2} = \frac{33}{8} = 4 \frac{1}{8} \]

(b) Simplify \( \frac{1}{(3 - x)} + \frac{5}{(2x + 4)} \)

Ans:
\[
\frac{1}{(3-x)} + \frac{5}{(2x+4)} = \frac{(2x+4) + 5(3-x)}{(3-x)(2x+4)} = \frac{2x + 4 + 15 - 5x}{(3-x)(2x+4)} = \frac{19 - 3x}{(3-x)(2x+4)}
\]

12. The graphs of the line \( y = 9 - x \) and \( y = 2x + 3 \) are shown below.

Use the graphs to find the solution to the simultaneous equations
\( y = 2x + 3 \) and \( y = 9 - x \)

Ans:
Looking at the graph, the solution is: \( x = 2 \) and \( y = 7 \)
13. (a) Work out the value of $\frac{5 \times 10^{-8}}{4 \times 10^{6}}$ giving your answer in standard index form.

**Ans:**

$$\frac{5 \times 10^{-8}}{4 \times 10^{6}} = \frac{5}{4} \times 10^{-8-6} = 1.25 \times 10^{-14}$$

(b) Jupiter is the largest planet in the Solar System.

The diameter of Jupiter is 11.2 times the diameter of the Earth.

The diameter of the Earth is $1.2756 \times 10^{4}$ km.

Work out Jupiter’s diameter, giving your answer in standard index form and correct to 3 significant figures.

**Ans:**

Jupiter’s diameter

$$= 11.2 \times 1.2756 \times 10^{4} = 14.28672 \times 10^{4} = 14.3 \times 10^{4} = 1.43 \times 10^{5} \text{ km}$$

14. Find the angle $x^0$ in the shape shown below.
A small loaf can be modelled in the shape of a cylinder, radius 3.5 cm and length 16 cm.
To make the loaf, 480 g of flour are used.
(a) What is the volume of the loaf?
\[ V = \pi r^2 h = \pi (3.5)^2 \times 16 = 615.75 \text{ cm}^3 \]

A larger loaf is also made. All the dimensions are increased by the same proportion.
The new loaf is 20 cm long.
(b) How much flour would be needed to make a similar milk loaf of length 20 cm?
\[ \text{The amount of flour} = \left( \frac{20}{16} \right)^3 \times 480 = 937.5 \text{ g} \]
(a) Calculate the angle that Janet’s string makes with the horizontal.

**Ans:**

\[
\cos A = \frac{b^2 + c^2 - a^2}{2bc} = \frac{162^2 + 126^2 - 178^2}{2 \times 162 \times 126} = 0.2556
\]

\[A = \cos^{-1}(0.2556) = 75.1909 \approx 75.2^\circ\]

(b) Hence calculate the height of the kite above the ground.

**Ans:**

\[
\sin A = \frac{h}{126}
\]

\[h = 126 \times \sin A\]
\[ h = 126 \times \sin\left(75.2^0\right) = 126 \times 0.9668233 = 121.81975 = 122 \text{ m} \]

17. Abdul Karim is paid £5.42 per hour for his work.

(a) Abdul Karim works out that to buy the computer game, he will have to work for exactly 10 hours.

How much does the computer game cost?

Ans: Cost of the game \( = 10 \times £5.42 = £54.2 \)

(b) Another computer game is £2.71 cheaper than the first one.

How much does the 2\textsuperscript{nd} computer game cost?

Ans: Cost of the 2\textsuperscript{nd} game \( £54.2 - £2.71 = £51.49 \)

(c) Raz takes home £98.90 per week for his job. He receives a 10% pay rise.

How much does Raz now take home?

Ans:
\[
\text{Raz now takes} = 98.90 + \frac{10}{100} (98.90) = 98.90 + 9.89 = £108.79
\]

OR
\[
\text{Raz now takes} = 1.1 \times 98.90 = £108.79
\]

18. (a) Express 12 as the product of its prime factors.

Ans:
\[
12 = 2 \times 2 \times 3 = 2^2 \times 3
\]

(b) Find the common lowest multiple of 12 and 15.

Ans:
\[
12 = 2 \times 2 \times 3 = 2^2 \times 3
\]
\[
15 = 3 \times 5
\]

\[ \text{LCM} = \text{Lowest Common Multiple} = LCM = 2^2 \times 3 \times 5 = 4 \times 3 \times 5 = 60 \]
13. In the expressions below, the letters a, b and c represent lengths. Tick the appropriate boxes to identify what each expression represents.

<table>
<thead>
<tr>
<th></th>
<th>$5\pi a^3$</th>
<th>$4(a^2 + b^2)$</th>
<th>$\frac{a^2 b^2}{c}$</th>
<th>$6(a + b + c)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length</td>
<td>✓</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Area</td>
<td></td>
<td>✓</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Volume</td>
<td>✓</td>
<td>✓</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

14. (a) Find the value of $a$ in the equation:

$3^a = \sqrt{27}$

**Ans:**

$3^a = \sqrt{27}$

$3^a = \sqrt[3]{3^3} = (3)^{\frac{3}{3}}$

$3^a = (3)^{\frac{3}{3}}$

$a = \frac{3}{2} = 1.5$

(b) Triangle XYZ has an area of $6cm^2$.  

Calculate $k$. Give your answer as a surd in its simplest form.

Ans: Area of the triangle $= \frac{1}{2}(k \times \sqrt{2}) = 6$

\[
\frac{\sqrt{2}k}{2} = 6
\]

\[
k = \frac{2 \times 6}{\sqrt{2}} = \frac{12\sqrt{2}}{2} = 6\sqrt{2}
\]

15. (c) Write down the equation of the line which is parallel to $y = \frac{1}{2}x$ and goes through the point $(0, -2)$.

Ans: $y = mx + b$

$m = \frac{1}{2}$, because it is parallel to $y = \frac{1}{2}x$
\[ y = \frac{1}{2} x + b \]

\[ -2 = 0 + b \implies b = -2 \]

\[ y = \frac{1}{2} x - 2 \]

The line described by this equation crosses the x-axis at the point \( k \).

(d) Calculate the coordinate of \( k \).

Ans:

\[ y = \frac{1}{2} x - 2 \]

When it crosses x-axis at \( k \), then \( y = 0 \) and \( x = k \)

\[ 0 = \frac{1}{2} k - 2 \]

\[ \frac{1}{2} k = 2 \implies k = 4 \]

Coordinates of \( k = (4,0) \)

16. Alex has £12 to spend on his little brother’s birthday.
He spends \( \frac{2}{5} \) of this money on sweet and \( \frac{1}{3} \) of this money on a story book.

(a) Work out how much money alex has left after he buys the sweet and the story book.

Ans:

Money spent on sweet \[ \frac{2}{5} \times 12 = \frac{24}{5} = £4.8 \]

Money spent on story book \[ \frac{1}{3} \times 12 = \frac{12}{3} = £4 \]

Money left over \[ 12 - (4.8 + 4) = 12 - 8.8 = £3.20 \]

OR

\[ \frac{2}{5} = \frac{6}{15} \]
\[ \frac{1}{3} = \frac{5}{15} \]
\[ \frac{6}{15} + \frac{5}{15} = \frac{11}{15} \]
\[
1 - \frac{11}{15} = \frac{15}{15} - \frac{11}{15} = \frac{4}{15}
\]

Money left over = \(\frac{4}{15} \times 12 = \frac{48}{15} = £3.20\)

Alex buys a 150-piece jigsaw puzzle. His brother completes 84 pieces of the puzzle.

(b) Calculate the percentage of the puzzle that Alex’s brother has completed.

\[
\text{Ans: } \frac{84}{150} \times 100 = 56\%
\]

17. One year, when Ahmad went on holiday to Greece, the exchange rate was £1 = 1.40 euros.

(a) Use the graph below to find out how many euros Ahmad got for £4.70.

\[
\text{Ans: From the graph below, there would be 6.70 euros for £4.70}
\]
The next year, the exchange rate had changed to £1 = 1.6 euros.

Draw a line on the grid above that could be used to convert money at this new rate.

Note: points: (0, 0), (1, 1.6), (2, 3.2), (3, 4.8), i.e. the line should pass through these points.
18. (a) Solve the following equations:

(i) \( \frac{29}{y} = 3 \)

Ans:

\[ \frac{29}{y} = 3 \]
\[ 3y = 29 \]
\[ y = \frac{29}{3} \]

(ii) \( 9y + 2 = 4(y - 3) \)

Ans:

\[ 9y + 2 = 4(y - 3) \]
\[ 9y + 2 = 4y - 12 \]
\[ 9y - 4y = -12 - 2 \]
\[ 5y = -14 \]
\[ y = -\frac{14}{5} = -4y = -2.8 \]

(b) Factorise \( 12a^2 - 8ab \)

Ans:

\[ 12a^2 - 8ab \]
\[ 4a(3a - 2b) \]

(c) Solve, by factorising, the equation \( 2x^2 - x - 6 = 0 \)

Ans:

\[ 2x^2 - x - 6 = 0 \]
\[ (2x + 3)(x - 2) = 0 \]

If \((2x + 3) = 0 \Rightarrow x = -\frac{3}{2} = -1.5\)

If \((x - 2) = 0 \Rightarrow x = 2\)

19. The table shows the heights and foot lengths of 10 girls.

<table>
<thead>
<tr>
<th>Height in m</th>
<th>1.60</th>
<th>1.75</th>
<th>1.56</th>
<th>1.65</th>
<th>1.68</th>
<th>1.48</th>
<th>1.79</th>
<th>1.75</th>
<th>1.61</th>
<th>1.72</th>
</tr>
</thead>
<tbody>
<tr>
<td>Foot length in cm</td>
<td>22</td>
<td>22.5</td>
<td>20.5</td>
<td>24.6</td>
<td>23</td>
<td>20.2</td>
<td>24.4</td>
<td>25.3</td>
<td>21</td>
<td>23.4</td>
</tr>
</tbody>
</table>

(a) Draw a scatter graph and a line of best fit to represent this information.
(b) Describe the correlation between height and foot length.

**Ans:**
Good positive correlation OR as the height increases so does the foot length and vice versa.
(c) Estimate the foot length of a girl who is 1.70 m tall.

**Ans:**
Read from the line on the graph. Answer between 23.0 cm and 23.8 cm are okay.

20. The grid below shows the graphs of the equations $2x - y = 1$ and $3x + 2y = 12$.

(a) Use the graphs to solve the simultaneous equations

\[
2x - y = 1 \quad \quad \quad 3x + 2y = 12
\]
\[ 2x - y = 1 \]
\[ 3x + 2y = 12 \]
\[ 2y + x = 4 \]

**Ans:**
\[ x = 2 \text{ and } y = 3 \] is the solution.

(b) On the same grid, draw the graph of \[ 2y + x = 4 \].

**Ans:**
See the graph.

(c) Shade the region represented by the inequalities \[ 2x - y \leq 1, \ 3x + 2y \leq 12 \] and \[ 2y + x \geq 4 \]. Label the region R.
21. Here is a histogram showing the speeds of cars on the A590.
(a) How many cars were travelling between 60 mph and 70 mph?

Ans: Area of ’60 - 70 mph’ = 10 × 2.4 = 24, so 24 cars were travelling between 60 mph and 70 mph.

(b) There were 16 cars travelling between 40 mph and 60 mph. Fill in the missing bar.

Ans: The area representing 16 cars must be 16. The width of the ’40 - 60 mph’ bar is 20, so the height must be \( \frac{16}{20} = 0.8 \).

(c) How many cars were surveyed altogether?

Ans: Total area = \( (40 \times 0.4) + (20 \times 0.8) + (10 \times 2.4) + (10 \times 2.8) = 16 + 16 + 24 + 28 = 84 \).

22. Make \( y \) the subject of this formula

\[
x = \frac{y^2 - 4}{3}
\]

Ans:

\[
3x = y^2 - 4
\]

\[
y^2 = 3x + 4
\]
\[ y = \sqrt{3x + 4} \]

23.

In the diagram, angles QSR and PQR are right angles. PQ = 7.5 cm, QS = 6 cm and SR = 8 cm.

(a) Calculate QR.

Ans:

\[ QR = \sqrt{6^2 + 8^2} = \sqrt{36 + 64} = \sqrt{100} = 10 \text{ cm} \]

(b) Calculate the value of \( \tan QPR \) and write your answer as a fraction in its simplest form.

Ans:

\[ \tan(QPR) = \frac{\text{Opposite}}{\text{Hypotenuse}} = \frac{QR}{PQ} = \frac{10}{7.5} = \frac{100}{75} = \frac{4}{3} \]

24. The durations of the first 20 songs played on the 107.3 Little FM radio station are shown on the cumulative frequency diagram below.
(a) Find the median song length.

**Ans:**

The median song length is given by \( \frac{1}{2} (n+1) = \frac{1}{2} (20 + 1) = \frac{21}{2} = 10.5 \), so from the graph: Median song length = 2.9 minutes
(b) Find the interquartile range of the song lengths.

The lower quartile position is given by: \( \frac{1}{4} (n + 1) = \frac{1}{4} (20 + 1) = \frac{21}{4} = 5.25^{th} \) 
value = 2.6 minutes

The upper quartile position is given by:
\( \frac{3}{4} (n + 1) = \frac{3}{4} (20 + 1) = \frac{63}{4} = 15.75^{th} \) value = 3.5 minutes

Range = 3.5 - 2.6 = 0.9

(c) Give one reason why the interquartile range is a better measure of spread than the range for this example.

The upper quartile position is given by:
\( \frac{3}{4} (n + 1) = \frac{3}{4} (20 + 1) = \frac{63}{4} = 15.75^{th} \) value = 3.5 minutes

25. (a) Expand and simplify:

\((x + 5)(2x - 1)\)

Ans:
\((x + 5)(2x - 1) = 2x^2 - x + 10x - 5 = 2x^2 + 9x - 5\)
(b) Factorise completely:

\[ 3x^2 - 6xy + 3x \]

**Ans:**

\[ 3x^2 - 6xy + 3x = 3x(x - 2y + 1) \]

26. Here is a plan of a garden. A 20 m rope attached to the building at point T stops the dog at the other end from straying too far. Construct accurately and shade the area of garden which can be reached by the dog.

![Garden Diagram](image)

27. The graph of \( y = f(x) \) is shown below:

![Graph of y = f(x)](image)

(a) Describe the transformations that would map the graph of \( y = f(x) \) onto the graph of:

(i) \( y = f(x + 7) \)

**Ans:**

A translation of \( \begin{bmatrix} -7 \\ 0 \end{bmatrix} \) i.e. 7 units in the negative x-direction as follows:
(b) The graph of which function is obtained by reflecting \( y = f(x) \) in the x-axis?

(c) The minimum point of \( y = 2x^2 \) is (0, 0).

What are the coordinates of the minimum point of \( y + 6 = 2(x - 3)^2 \)?

28. There has been a problem on a car production line, and all the cars need to be recalled for a check. The probability that one of these cars has faulty steering is 0.1 and the probability that it has a faulty handbrake is 0.2. These probabilities are independent of each other.

(a) Fill in the probabilities on the tree diagram below.
Using the answer to (a), or otherwise, find the probability that one of these cars has:

(iii) both fault.

Ans:

0.1\times 0.2 = 0.02

(iv) neither fault.

Ans:

0.9\times 0.8 = 0.72

(iii) only one fault.

Ans:

\[(0.1\times 0.8) + (0.9\times 0.2) = 0.08 + 0.18 = 0.26\]
29. In the diagram below, line ABC is tangent to the circle, centre O. Angle CBD is $48^\circ$.

(a) Calculate the size of the angle OBD.
$$OBD = 90^\circ - 48^\circ = 42^\circ$$

(b) Calculate the size of the angle BOD.
$$BOD = 180^\circ - 42^\circ - 42^\circ = 96^\circ$$

(c) Calculate the size of the angle BED.
$$BED = \frac{1}{2} \times BOD = \frac{1}{2} \times 96^\circ = 48^\circ$$

Narrated Anas (RA): The Prophet(SAW) used to say, "O Allah! Our Lord! 
Abdulh and Wardak

Give us in this world that, which is good and in the Hereafter that,
Which is good and save us from the torment of the Fire” (Al-Bukhari)

Dear Brothers and Sisters!
I kindly request each one of you to remember me, my parents, my children, my family and the entire Muslims of the world in your daily Prayers. **If you have not done it so far, then please do it now.**

السلام عليكم و رحمت الله و بركاته!

I request you all to remember me, my parents, my children, my family and the entire Muslims of the world in your daily Prayers.

وينسي الذين لم يذكروا، **أطلب منكم أنتم جميعًا، أن تذكرواني، والهندس، أولادك، والكل منكم مع همّةٍ ودومًا.**

السلام، عبد الله وردك، عبد الله وردك.

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